Segédlet az EM terek többdimenziós numerikus modellezéshez

(Hosszan elnyúlt szerkezetek esetén kialakuló EM terek frekvenciatartománybeli numerikus modellezése)

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EM terek numerikus modellezése

Bevezetés

A numerikus modellezésre azért van igény, mert a legtöbb esetben kialakuló EM terek csak ritkán kezelhetők, ill. adhatók meg analitikus, zárt formában. Utóbbi megoldások csak egyszerűbb forrástér feltételezése mellett, homogén, izotróp tulajdonságú közegek (általában homogén féltérre vagy rétegzett féltérre, amelyek 1D-s szituációk) esetén alkalmazhatók. A több dimenziós feladatok megoldásához azonban numerikus módszerekre és ezzel együtt megfelelő számítógépes háttérre van szükség, ahol vagy a teljes EM térre vagy csak annak szekunder részére végzünk számításokat. Az EM tér matematikailag akkor válik kezelhetővé, ha a Maxwell – egyenletekből kiindulva a leszármaztatott egyenleteket parciális differenciálegyenletekké (DE), vagy integrálegyenletekké (IE) alakítjuk. Ezt követően a DE egyenleteket megoldhatjuk a véges különbségek módszerével (finite difference method, FD), vagy a variációs elv alkalmazásával a véges elemek módszerével (finite element method, FE). Az integrálegyenletek módszere a harmadik legfontosabb eljárás. Gyakran a különböző numerikus eljárások kombinációját alkalmazzuk, ilyenkor hibrid módszerekről beszélünk. Az említett eljárások az EM több dimenziós inverziós feladat alapelemei is lehetnek, ugyanis az inverzió során az előre modellezési feladat többszöri megoldására kerül sor. A három legfontosabb eljárás lényegének az ismertetésére a következőkben kerül sor.

A véges különbségek módszerénél a kiindulás a forrástaggal bővített Maxwell – egyenletek. Síkhullámú esetben a Maxwell – egyenletekből a távíróegyenlet származtatható le, és harmonikus időfüggés feltételezve a távíró egyenletek időtől független Helmholtz – egyenletekké alakulnak át. A hullámegyenletekben a deriváltak többváltozós Taylor sorfejtés révén közelíthetők. A mesterséges áramterű forrás esete bonyolultabb, és a feladat megfogalmazását jelentő parciális differenciálegyenlet(ek) is összetettebb(ek). A megoldás elve azonban közös, mert mindkét esetben a modellezni kívánt tértartományra egy derékszögű négyszöghálót fektetünk, és ezen háló rácspontjaiban a megoldás közelítő értékét határozzuk meg úgy, hogy minden egyes rácspontra az ismeretlen függvény értékét a szomszédos pontokhoz tartozó értékek lineáris kombinációjaként írjuk fel. Leggyakrabban 5- vagy 9-

pontos rácsmódszert alkalmaznak. A feladat és a modell jellegétől függő nagy méretű lineáris rendszert kell megoldani a matematikai értelemben vett elsődleges komponens(ek)re. Mint a többi numerikus eljárásnál itt is érvényesíteni kell a határfelületekkel párhuzamos térkomponensek folytonosságát a megfelelő peremfeltételek figyelembe vétele mellett. Ennek megfelelően a véges differenciaegyenletek eltérő alakúak.

A véges elemek módszere rugalmasabb, mint a véges különbségek módszere, ugyanis tetszőleges geometriájú sík vagy térbeli rácsra alkalmazható. Itt a differenciaegyenletek levezetése a variációs elvek felhasználásával történik, melyek az együtthatómátrix szimmetriáját és pozitív definit tulajdonságát is biztosítják. Itt tehát nem a Taylor sorfejtéses technikát alkalmazzuk, így filozófiájában is eltér a véges különbséges módszertől. Az EM módszerek vonatkozásában Coggon és Ryu (1971) alkalmazták először, ahol a rendszert leíró függvény a Poynting vektorral jellemezhető. Az elektromágneses módszerekre a teljes potenciális energiát a Maxwell egyenletekből kiindulva a Poynting teoréma alapján lehet meghatározni. A variációs integrál a síkhullámú, a mágneses vagy elektromos forrást feltételező esetre megadható, mely az Euler – Lagrange egyenlet alkalmazásával a megfelelő differenciál egyenletté alakítható át. Az energiafüggvény minimalizálása közelítőleg elvégezhető az egyes véges elemeken felvett lineáris interpolációs függvények segítségével. A megoldás függvényt a szerkezethez jobban alkalmazkodó szabálytalan rács elemek belső pontjaiban vagy a rácspontokban adják meg. Lényeges kérdés a rácspontok sorszámozása, mely az együtthatómátrix struktúráját befolyásolja. A cél itt is a minimális sávszélességű együtthatómátrix kialakítása a lineáris algebrai feladat megoldása szempontjából. A véges elemes módszer alkalmazása előnyt élvez a véges különbséges módszerekkel szemben különösen akkor, ha dőlt vagy görbült határfelületek jellemzik a modellünket.

A numerikus modellezés harmadik lehetséges változata az integrálegyenletek módszere. A Maxwell egyenletek a Gauss -, illetve Stokes – tétel alkalmazásával integrál alakra hozható, majd ezen integrálegyenletek mátrix – egyenletekké alakíthatók át. A módszer fontos jellemzője, hogy az inhomogenitásokon kívüli térrészben az EM tér meghatározható az inhomogenitásokat körülzáró felületre vonatkozó integrálok segítségével. Ezen integrálok Green függvényeket és fiktív elektromos és mágneses felületi áramsűrűségét, továbbá elektromos és mágneses felületi töltéssűrűséget tartalmaznak. A feladat itt is az előző két eljáráshoz hasonlóan lineáris rendszer megoldására vezet. Ez a módszer elsősorban akkor ajánlható ha 1D – s környezetben lévő, kis számú több dimenziós szerkezet hatását kívánjuk elemezni. Ilyenkor a feladat ezen eljárással lényegesen kisebb CPU idővel és kisebb kapacitású számítógéppel oldható meg, mint a totális EM teret számító véges elemes vagy véges különbséges módszerrel.

Ebben a jegyzetben főleg magyar de angol nyelvű szemelvények formájában is kaphatunk áttekintést a témához kapcsolatosan.

FREKVENCIA TARTOMÁNYBELI EM MÓDSZEREKHEZ ELMÉLETI ALAPOK

A telegráf egyenlet az EM tér $e^{+i\varpi t}$ időszerinti változását feltételezve a következő alakú (a telegráf egyenlet levezetését az 1. függelék tartalmazza), melyben jelen esetben csak az $\vec{E}$ elektromos térerősség vektor szerepel (teljesen hasonló az egyenlet a mágneses térvektor vonatkozásában):

$$\Delta\vec{E} + (\mu\varepsilon\varpi^2 - i\varpi\mu\sigma)\vec{E} = \Delta\vec{E} + k^2\vec{E} = \vec{0} \tag{1}$$

A fenti egyenlet megoldása akkor ha csak E_x komponenst tételezünk fel és a hullám lefelé terjed a homogén izotróp féltérben:

$$E_x(z,t) = E_{xo}e^{-ikz}e^{i\varpi t} = E_{xo}e^{-i\alpha z}e^{-\beta z}e^{i\varpi t} \tag{2}$$

Itt azt fogalmaztuk meg, hogy $k = \alpha - i\beta$ alakú. Így a

$$k^2 = \alpha^2 - 2i\alpha\beta - \beta^2 = \mu\varepsilon\varpi^2 - i\varpi\mu\sigma \tag{3}$$

egyenletből a valós és képzetes részek egyenlőségéből α és β meghatározható. A gyökvonásoknál csak a pozitív értékeket figyelembe véve írható, hogy

$$\alpha = \varpi\left\{\frac{\mu\varepsilon}{2}\left[\left(1+\frac{\sigma^2}{\varepsilon^2\varpi^2}\right)^{\frac{1}{2}}+1\right]\right\}^{\frac{1}{2}} \quad \text{és} \quad \beta = \varpi\left\{\frac{\mu\varepsilon}{2}\left[\left(1+\frac{\sigma^2}{\varepsilon^2\varpi^2}\right)^{\frac{1}{2}}-1\right]\right\}^{\frac{1}{2}} \tag{4}$$

A szkín mélység az a mélység, melyben a felszíni térérték az *e*-ad részére csökken. A (2) egyenlet jobb oldalán az amplitúdó mélységgel való csökkenését $E_{xo}e^{-\beta z}$ írja le, így a szkín mélység - z_s - számításához az alábbi egyenletet kell megoldani:

$$E_{xo}e^{-\beta z_s} = E_{xo}e^{-1} \text{ ebből} \qquad z_s = 1/\beta = 1/\varpi\left\{\frac{\mu\varepsilon}{2}\left[\left(1+\frac{\sigma^2}{\varepsilon^2\varpi^2}\right)^{\frac{1}{2}} - 1\right]\right\}^{\frac{1}{2}} \tag{5}$$

Tehát a β csillapítási tényező és a szkín mélység egymással fordítottan arányos.
A hullámhossz az azonos fázisú pontok közötti távolság. A (2) egyenlet jobb oldalán a fázisviselkedést a második tényező írja le, így λ az

$e^{-2\pi i} = e^{-\alpha\lambda i}$ feltételből határozható meg, amiből

$$\lambda = 2\pi/\alpha = 2\pi/\varpi\left\{\frac{\mu\varepsilon}{2}\left[\left(1+\frac{\sigma^2}{\varepsilon^2\varpi^2}\right)^{\frac{1}{2}} + 1\right]\right\}^{\frac{1}{2}} \tag{6}$$

A reflexiós szeizmika és a reflexiós georadar módszerek hasonlósága alapján állítható, hogy a georadar módszer vertikális felbontóképessége (az a rétegvastagság mely mellett a vizsgált réteg felső és alsó határfelületéről kapott reflexió egymástól megkülönböztethetően elválik, így a réteg ″kimutatható″a reflexiós időszelvényen) $\lambda/4$, azaz a hullámhossz függvényében legalább ilyen vastagnak kell lenni a rétegnek (Rayleigh-kritérium). A laterális felbontóképesség mértékét az adó-vevő távolság (csökkentése a laterális felbontóképesség növelését segíti elő) és a Fresnel zóna nagysága határozza meg. amely a domináns hullámhossz és a vizsgált objektum reflektáló felülete mélysége szorzatának négyzetgyökével arányos akkor, ha a reflektáló felület mélység szintje sokkal nagyobb, mint a hullámhossz.
A Fresnel zónára vonatkozó összefüggésből meghatározható az a laterálisan jelentkező méret, amely még kimutatható. Tehát mind a vertikális, mind a horizontális felbontás mértéke függ a hullámhossztól (a Fresnel zónára vonatkozó levezetést a 2. sz függelék tartalmazza).
A lefelé terjedő síkhullám terjedési sebessége $v = \lambda/\mathrm{T} = \lambda f$, így

$$v = 1/\left\{\frac{\mu\varepsilon}{2}\left[\left(1+\frac{\sigma^2}{\varepsilon^2\varpi^2}\right)^{\frac{1}{2}} + 1\right]\right\}^{\frac{1}{2}} \tag{7}$$

Az összefüggések az eltolási áramok elhanyagolásával élő esethez képest tehát bonyolultabbak, emlékeztetőül ott

$$\alpha = \beta = \left[\frac{\varpi\mu\sigma}{2}\right]^{\frac{1}{2}} \text{ miatt} \qquad z_s = \left[\frac{2}{\varpi\mu\sigma}\right]^{\frac{1}{2}} \qquad \lambda = 2\pi\left[\frac{2}{\varpi\mu\sigma}\right]^{\frac{1}{2}} \tag{8}$$

voltak, és ebben az estben az EM hullám homogén vezetőképességű féltérbeli terjedési sebessége:

$$v = \lambda / T = 2\pi\left[\frac{f}{\pi\mu\sigma}\right]^{\frac{1}{2}} \tag{9}$$

Másik határesetet akkor kapjuk meg, ha azt feltételezzük, hogy az eltolási áramok mértéke sokszorosa a vezetési áramokéhoz képest. Ez az eset alacsony vezetőképesség mellett, egészen nagy frekvenciájú EM tereknél jelentkezik. Ekkor (1) egyenlet a következő alakú:

$$\Delta\vec{E} + \mu\varepsilon\varpi^2\vec{E} = \vec{0} \tag{10}$$

(3) szerinti felbontást figyelembe véve

$$\alpha = \sqrt{\mu\varepsilon\varpi^2} \text{ és } \beta = 0 \tag{11}$$

azaz ez EM térnek csillapodása nincs, a hullámhossz pedig (6) előtti feltételből

$$\lambda = 2\pi / \alpha = 2\pi / \sqrt{\mu\varepsilon\varpi^2} = \frac{1}{f\sqrt{\mu\varepsilon}} = \frac{v}{f} \tag{12}$$

amiből a terjedési sebesség

$$v = \frac{1}{\sqrt{\mu\varepsilon}} = \frac{1}{\sqrt{\mu_o\varepsilon_o\varepsilon_r}} = \frac{c}{\sqrt{\varepsilon_r}} \tag{13}$$

ahol c a fény terjedési sebessége levegőben. (12) és (13) felhasználásával a hullámhossz ekkor közelíthető

$$\lambda = \frac{c}{f\sqrt{\varepsilon_r}} \tag{14}$$

értékkel. Ebből a formulából az következik, hogy a **frekvencia növelésével a hullámhossz csökken, így a felbontóképesség nő,** illetve az is látszik, hogy **a relatív dielektromos állandó növekedése szintén a jobb felbontást eredményező tényezők egyike. A felbontóképességet döntően befolyásoló ezen két tényező mellett megállapítható a hullámhossz csökkenése a vezetőképesség növekedése mellett a kis frekvenciás, ill. az általános esetben is** (8) és (6) szerint.

Az EM tér csillapodását a különböző esetekre (8)- , (4)- és (11)-ben szereplő β csillapítási tényező írja le. A felszín alatti tértartományban mindig fellép az EM tér gyengülése és a georadarnál alkalmazott frekvenciák mellett (4) jobb oldali összefüggése lehet a kiindulás.
Ha $\sigma^2/\varepsilon^2\varpi^2 << 1$, akkor $(1+\sigma^2/\varepsilon^2\varpi^2)^{1/2} \approx 1+\sigma^2/2\varepsilon^2\varpi^2$ alapján írható (4) helyett

$$\beta = \varpi\left\{\frac{\mu\varepsilon}{2}\left[\left(1+\frac{\sigma^2}{\varepsilon^2\varpi^2}\right)^{\frac{1}{2}}-1\right]\right\}^{\frac{1}{2}} \approx \varpi\left\{\frac{\mu\varepsilon}{2}\left[1+\frac{\sigma^2}{2\varepsilon^2\varpi^2}-1\right]\right\}^{\frac{1}{2}} = \frac{1}{2}\left(\frac{\mu}{\varepsilon}\right)^{\frac{1}{2}}\sigma \qquad (15)$$

Tehát **a csillapítási tényező az elektromos vezetőképességgel lineárisan, míg a relatív dielektromos állandóval fordítottan arányos abban az esetben amikor mind a vezetési, mind az eltolási áramok hatását figyelembe vesszük, azonban az eltolási áramok hatása lényegesen nagyobb.** A fenti feltételezés mellett a csillapítási tényező frekvencia szerinti függése nem jelentkezik, tehát ebben a frekvencia tartományban nem a frekvenciális függés a meghatározó mindaddig , míg $\sigma^2/\varepsilon^2\varpi^2 << 1$, azonban ez a hányados még nem tart a zérushoz.
Ugyanakkor **a csillapítási tényező a frekvencia növelésével** rögzített σ és ε_r esetén (4) jobb oldali egyenlete szerint **nő**, kis frekvenciák mellett (8) használható.
A georadar méréseknél a kimutatandó objektum mérete és a hullámhossz kapcsolata mellett fontos tehát az, hogy a kimutatandó objektum mélység szintjében még megfelelően nagy legyen az EM jel energiája. Ugyanakkor az is fontos, hogy a megfelelően nagy beérkező jel visszaverődjön. Ennek feltétele, hogy az egymással érintkezésben lévő anyagok elektromágneses paraméterei között kimutathatóságot lehetővé tevő eltérés legyen.
Vízszintesen 2 réteges féltér esetére rétegződésre merőleges síkhullámú EM hullámterjedést feltételezve a reflexiós tényező (*R)* értéke

$$R = \frac{Z_2 - Z_1}{Z_2 + Z_1} \qquad (16)$$

ahol Z a síkhullámú EM tér impedanciája, az 1-es a felső, a 2-es index az alsó rétegre vonatkozó jelölés. Síkhullámú terekre érvényes impedancia összefüggés az $e^{+i\varpi t}$ időszerinti változását feltételezve felírt $rot\vec{E} = -\partial\vec{B}/\partial t = -i\varpi\mu\vec{H}$ Maxwell-egyenletből származtatható le úgy, hogy homogén, izotróp féltérben lefelé haladó, E_x és H_y komponensekkel jellemezhető síkhullámot tételezünk fel. Az *y* irányú komponensre felírt Maxwell-egyenletből :

$$\frac{\partial E_x}{\partial z}\vec{j} = -i\varpi\mu H_y\vec{j} \qquad (17)$$

Az x irányú elektromos térkomponens z szerinti derivált értéke (2) alapján $\partial E_x / \partial z = -ikE_x$. Ezt (17)-be helyettesítve, majd az egyenletet az impedancia értékére rendezve kapjuk, hogy

$$Z_{xy} = \frac{E_x}{H_y} = \frac{\varpi\mu}{k} \tag{18}$$

Könnyen belátható $\mu_1 = \mu_2 = \mu_o\mu_r = \mu_o$, ill. $k_i^2 \approx \mu_0\varepsilon_i\varpi^2$ feltétezésekkel élve, hogy (16) helyett

$$R = \frac{\frac{\varpi\mu_2}{k_2} - \frac{\varpi\mu_1}{k_1}}{\frac{\varpi\mu_2}{k_2} + \frac{\varpi\mu_1}{k_1}} \approx \frac{\sqrt{\varepsilon_{r1}} - \sqrt{\varepsilon_{r2}}}{\sqrt{\varepsilon_{r1}} + \sqrt{\varepsilon_{r2}}} \tag{19}$$

írható. Összefoglalva a nagyobb frekvenciákon **a reflektált jel amplitúdója elsősorban az érintkező közegekre jellemző relatív dielektromos állandók négyzetgyöke közötti különbségtől függ, azzal lineárisan arányos.**

2D MT véges különbséges modellezés

A megoldandó feladat azt tételezi fel, hogy a vizsgált szerkezet a mérési területen megnyúltsági iránnyal jellemezhető. Ezt a feltételezést általában más (pl.gravitációs vagy mágneses) módszerek mérési eredményei alapján tehetjük meg. Az anomáliák megnyúltsági iránya a szerkezeti csapásvonalat jelöli ki. Az általános eset természetesen 3D-s, azonban gyakran alkalmazható a 2D-s közelítés. A feladat kezelhetősége végett azonban azt feltételezzük, hogy a szerkezet a csapás irányában végtelen kiterjedésű, tehát ezen irányra merőleges síkok bármelyikében azonos geometriával és konduktivitás eloszlással jellemezhető metszetet kapnánk.

A 2D-s szerkezeti feltételezés mellett a további egyszerűsítésekkel, ill. feltételezésekkel élünk:

- A vizsgált tértartományban a beérkező EM tér megegyezik
- Egyetlen diszkrét f frekvenciájú, $e^{i\varpi t}$ harmonikus időfüggésű EM teret vizsgálunk
- A vezetési áram lényegesen nagyobb mint az eltolási áram
- A közeg izotróp
- Vízszintes a felszín

Először azt bizonyítjuk be, hogy ezen feltételezések mellett, a felszínre tetszőleges szöggel beeső EM síkhullám előállítható egy tisztán E módusú, ill. H módusú hullám szuperpozíciójaként.

Induljunk ki az I. Maxwell-egyenletből – mely szerint mind a vezetési áram, mind az eltolódási áram időben mágneses örvényteret létesít. Az eltolódási áramok hatásának elhanyagolásával:

$$rot\vec{H} = \vec{j} + \frac{\partial \vec{D}}{\partial t} \approx$$

$$\approx \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ H_x & H_y & H_z \end{vmatrix} = \vec{i}\left(\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z}\right) - \vec{j}\left(\frac{\partial H_z}{\partial x} - \frac{\partial H_x}{\partial z}\right) + \vec{k}\left(\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y}\right) = \sigma\left(\vec{i}E_x + \vec{j}E_y + \vec{k}E_z\right)$$

Írjuk fel a II. Maxwell-egyenletet, amely azt fejezi ki, hogy az időben változó mágneses tér elektromos örvényteret hoz létre (továbbá itt is végezzük el a komponensekre bontást is):

$$rot\vec{E} = -\frac{\partial \vec{B}}{\partial t} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x & E_y & E_z \end{vmatrix} = \vec{i}\left(\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z}\right) - \vec{j}\left(\frac{\partial E_z}{\partial x} - \frac{\partial E_x}{\partial z}\right) + \vec{k}\left(\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y}\right) =$$

$$= -i\varpi\mu\left(\vec{i}H_x + \vec{j}H_y + \vec{k}H_z\right)$$

Vegyük figyelembe, hogy a beeső EM tér állandó a vizsgált tértartományban, és a szerkezetnek csapásirányban EM tértorzító hatása nem lehet:

$$\frac{\partial E}{\partial x} = \frac{\partial H}{\partial x} = 0$$

Ez utóbbi megállapítást érvényesítve a komponens egyenletekre az I., ill. II. Maxwell-egyenletek helyett írható:

$$\left(\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z}\right)\vec{i} + \left(\frac{\partial H_x}{\partial z}\right)\vec{j} - \left(\frac{\partial H_x}{\partial y}\right)\vec{k} = \sigma E_x\vec{i} + \sigma E_y\vec{j} + \sigma E_z\vec{k}$$

$$\left(\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z}\right)\vec{i} + \left(\frac{\partial E_x}{\partial z}\right)\vec{j} - \left(\frac{\partial E_x}{\partial y}\right)\vec{k} = -i\varpi\mu H_x\vec{i} - i\varpi\mu H_y\vec{j} - i\varpi\mu H_z\vec{k}$$

Csoportosítást úgy végezzük el, hogy külön legyenek azok az egyenletek, amelyekben a csapásirányú elektromos térkomponens E_x szerepel, ill. azok melyekben a csapásirányú mágneses térkomponens H_x van. Az első Maxwell-egyenletből az $\vec{i}$ -re a másodikból a $\vec{j}$ -re, ill. $\vec{k}$ -ra vonatkozó egyenletekből

$$\left(\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z}\right) = \sigma E_x$$

$$\frac{\partial E_x}{\partial z} = -i\varpi\mu H_y$$

$$\frac{\partial E_x}{\partial y} = i\varpi\mu H_z$$

Jól látható, hogy ezen 3 egyenletben a csapásirányú elektromos , a dőlésirányú és vertikális mágneses térkomponens szerepel. Annak érdekében, hogy a matematikailag egyszerűbb esetet kapjunk, az utóbbi két egyenletből H_y és H_z értékét helyettesítsük be az előttük lévőbe:

$$\frac{\partial^2 E_x}{\partial y^2} + \frac{\partial^2 E_x}{\partial z^2} = i\varpi\mu\sigma E_x$$

Ezzel eljutottunk a Helmholtz-féle időtől független parciális differenciálegyenlethez, amelyben egyetlen ismeretlen -E_x- szerepel. A másik polarizáció esetén teljesen hasonlóan járunk el: megkeressük azon komponens egyenleteket, melyekben H_x található. Jól látható, hogy ezek éppen a megmaradt, eddig fel nem használt egyenletek. Tehát:

$$\left(\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z}\right) = -i\varpi\mu H_x$$

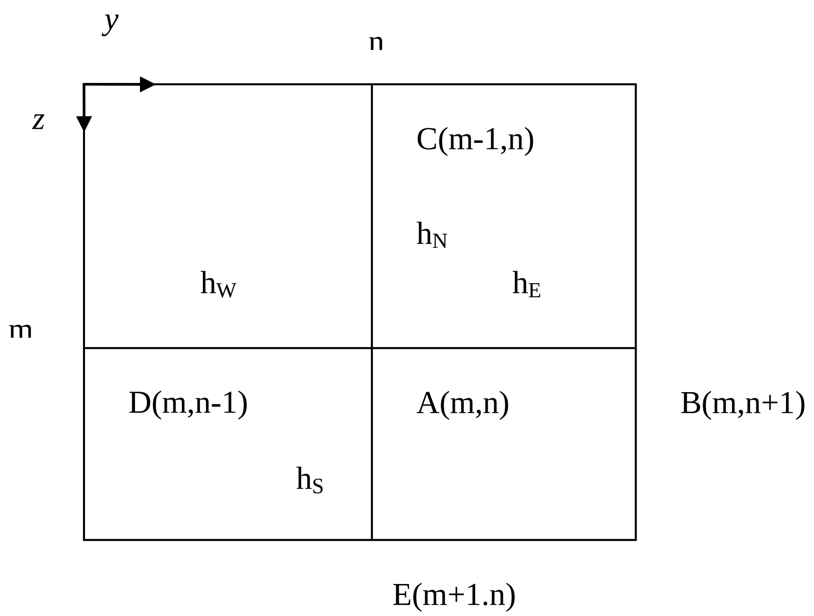
$$\frac{\partial H_x}{\partial z} = \sigma E_y$$

$$-\frac{\partial H_x}{\partial y} = \sigma E_z$$

Az egyenletek előzőhöz teljesen hasonló összedolgozásából kapjuk:

$$\frac{\partial^2 H_x}{\partial y^2} + \frac{\partial^2 H_x}{\partial z^2} = i\varpi\mu\sigma H_x$$

2D-s szerkezetek esetén harmonikus időszerinti térváltozást feltételezve tehát azt tapasztaltuk, hogy a tetszőleges polarizációjú síkhullám felbontható két, egymástól független eset szuperpozíciójára. Ez a külön válás indokolja, hogy a két polarizáció matematikailag is egymástól függetlenül tárgyalható. Homogén, izotróp tértartományban a két polarizáció megegyező alakú differenciálegyenlettel írható le. A következőkben nézzük meg, hogy inhomogenitás nélküli esetben milyen véges differencia egyenlettel közelíthető a $\Delta\phi + k^2\phi = 0$ alakú egyenlet, ahol ϕ skalár mennyiség, ui. csapásirányú (x irányú) EM térkomponens.



1.ábra

Ennek érdekében a kétváltozós függvények Taylor-sorfejtéses alakjából induljunk ki. Ha egy y,z koordinátájú pontban ismert a függvényérték, akkor egy $y+h_y$, $z+h_z$ koordinátájú pontban a függvény értéke az alábbiak szerint határozható meg:

$$\phi(y+h_y, z+h_z) = \phi(y,z) + \left(\frac{\partial\phi}{\partial y}h_y + \frac{\partial\phi}{\partial z}h_z\right) + \frac{1}{2}\left(\frac{\partial^2\phi}{\partial y^2}h_y{}^2 + \frac{\partial^2\phi}{\partial y\partial z}h_y h_z + \frac{\partial^2\phi}{\partial z^2}h_z{}^2\right)$$

Az 1. ábra tünteti fel a koordinátarendszer irányítottságát, m a sorok, n az oszlopok számát jelöli. Az A(m,n) pontbeli függvényérték ismeretében a B, C, D, E rácspontbeli függvényértékek a következők:

$$\phi(m,n+1) = \phi(m,n) + \frac{\partial\phi}{\partial y}h_E + \frac{1}{2}\frac{\partial^2\phi}{\partial y^2}h_E{}^2 \qquad \text{(B)}$$

$$\phi(m-1,n) = \phi(m,n) - \frac{\partial\phi}{\partial z}h_N + \frac{1}{2}\frac{\partial^2\phi}{\partial z^2}h_N{}^2 \qquad \text{(C)}$$

$$\phi(m,n-1) = \phi(m,n) - \frac{\partial\phi}{\partial y}h_W + \frac{1}{2}\frac{\partial^2\phi}{\partial y^2}h_W{}^2 \qquad \text{(D)}$$

$$\phi(m+1,n) = \phi(m,n) + \frac{\partial\phi}{\partial z}h_S + \frac{1}{2}\frac{\partial^2\phi}{\partial z^2}h_S{}^2 \qquad \text{(E)}$$

B egyenletet h_w-vel, D egyenletet h_E-vel szorozzuk meg azért, hogy a megszorzott egyenletek összeadása során az elsőrendű deriváltak kiessenek. Hasonlóan C egyenletet h_S-sel, míg E egyenletet h_N-nel szorozzuk meg majd összegezzük őket:

$$h_W\phi(m,n+1) + h_E\phi(m,n-1) = (h_W + h_E)\phi(m,n) + (h_W h_E{}^2 + h_E h_W{}^2)\frac{1}{2}\frac{\partial^2\phi}{\partial y^2}$$

$$h_S\phi(m,n+1) + h_N\phi(m+1,n) = (h_S + h_N)\phi(m,n) + (h_S h_N{}^2 + h_N h_S{}^2)\frac{1}{2}\frac{\partial^2\phi}{\partial z^2}$$

Az előbbi egyenletből a skalár függvény dőlés-, míg az utóbbiból a vertikális irány szerinti második derivált közelítő értéke fejezhető ki. Ezek ismeretében homogén esetre a Helmholtz

egyenlet véges különbséges alakja könnyen megadható, hisz a $\Delta\phi + k^2\phi = 0$ egyenletben a második derivált közelítő értéke így adott, míg a második tag értelemszerűen $k^2\phi(m,n)$ lesz.
A fentiek értelmében a véges differencia egyenlet

$$a\phi(m,n) + b\phi(m,n+1) + c\phi(m-1,n) + d\phi(m,n-1) + e\phi(m+1,n) = 0 \qquad \text{(F)}$$

alakú, ahol b, c, d, e együtthatók a frekvenciától és a konduktivitástól függetlenek, tehát csak a rács geometriai méreteitől függenek. Ez nem mondható el az a együtthatóról, mely a hullámszámnak –így mind a frekvenciának, mind a konduktivitásnak- függvénye. Fontos kihangsúlyozni, hogy mindez csak az elvi jelentőségű homogén esetre igaz. Másrészt az is nyilvánvaló, hogy ezen közelítésekkel egy 5 pontos rácsmódszer (véges különbséges) kiindulásának alapjait sikerült bemutatni. Az együtthatók tehát homogén tértartományra:

$$a = k^2 - 2\left(1/h_E\, h_W + 1/h_S h_N\right) \qquad \text{(G)}$$

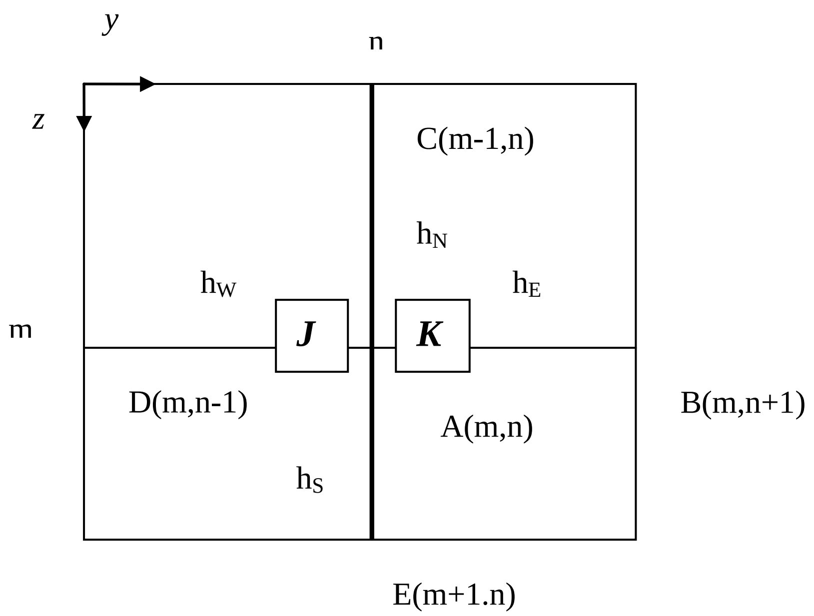
$$b = 2/(h_E + h_W)h_E \qquad c = 2/(h_S + h_N)h_N$$
$$d = 2/(h_E + h_W)h_W \qquad e = 2/(h_S + h_N)h_S \qquad \text{(H)}$$

Tehát ezek az együtthatók konstans hullámszámú hálórészre, a rács belsejében érvényesek.
Az eddigiek viszont szükségesek voltak ahhoz, hogy áttérjünk az egymással érintkezésben lévő, különböző konduktivitású térrészek esetére. A határfelületeken az elektromos és a mágneses térerősség tangenciális összetevői folytonosak. A határfelületek a derékszögű hálót alkalmazó véges különbséges modellezés során vízszintesek vagy függőlegesek lehetnek. Nézzük azt az esetet, amikor egy függőleges sík választ el két különböző konduktivitású térrészt a 2. ábra szerint. Tételezzük fel az E polarizáció esetét. Ekkor a C-, D-, E rácspontokra felírt egyenletek a ***J***-ik közegre vonatkoznak ugyanis a D pontra felírt Taylor-sorfejtéses formulában az A pontbeli érték épp az A pontbeli E_x érték, ill. a C és E rácspontokra vonatkozó egyenletekben valamennyi ϕ érték határfelületi tangenciális elektromos térkomponens érték (E_x). A cél az, hogy a ***K***-ik közegben lévő B pontbeli csapásirányú elektromos térkomponenst a ***J***-ik közegre vonatkozó csapásirányú elektromos térkomponens függvényében adjuk meg. A kiindulás értelemszerűen a B egyenlet:

$$E_x^{\,K}(m,n+1) = E_x^{\,K}(m,n) + \frac{\partial E_x^{\,K}}{\partial y} h_E + \frac{1}{2}\frac{\partial^2 E_x^{\,K}}{\partial y^2} h_E^{\,2}$$

A törekvésünk formálisan azt jelenti, hogy ezen egyenlet jobb oldalán valamennyi ***K*** index helyett ***J*** szerepeljen. Mivel ezen vertikális határfelület mentén E_x mellett H_z folytonossága is fenn áll, továbbá a vertikális mágneses térkomponens épp E_x dőlésszerinti változásával arányos (lásd az E polarizációra megadott 3. egyenlet) , ha a két közegre a relatív mágneses permeabilitás megegyezik, akkor $\partial E_x / \partial y$ is. A jobb oldalon lévő második derivált másik közegre vonatkozó kifejezése érdekében a két közegre egyaránt érvényes Helmholtz egyenlet egyenlőségének felírásából célszerű kiindulni.

$$\Delta E_x^{\,J} + k_J^{\,2} E_x^{\,J} = \Delta E_x^{\,K} + k_K^{\,2} E_x^{\,K} = 0$$



2.ábra

Ezen egyenletben a második derivált csak dőlésirányban marad meg, ugyanis a csapásirányú elektromos térkomponens folytonossága a határfelület mentén folyamatosan teljesül, így a z szerinti első és második deriváltakra is fennáll az egyenlőség. A ***K***-ik közegre vonatkozó dőlés szerinti második derivált:

$$\frac{\partial^2 E_x{}^K}{\partial y^2} = \frac{\partial^2 E_x{}^J}{\partial y^2} + \left(k_J{}^2 - k_K{}^2\right)E_x{}^J$$

Ezt is behelyettesítve a B egyenlet ezelőtti alakjába kapjuk, hogy

$$E_x{}^K(m,n+1) = E_x{}^J(m,n) + \frac{\partial E_x{}^J}{\partial y} h_E + \frac{1}{2} h_E{}^2 \left(\frac{\partial^2 E_x{}^J}{\partial y^2} + \left(k_J{}^2 - k_K{}^2\right)E_x{}^J\right)$$

Tehát ezzel célunkat elértük, így a ***K*** index használata feleslegessé vált, elértük azt, hogy az eltérő konduktivitású tértartományokat elválasztó vertikális határfelület esetén valamennyi Taylor –sorfejtéses közelítésben a baloldali térkomponensek és azok deriváltjai szerepelnek a két konduktivitás és a frekvencia értéke mellett. Ilyen modell feltevésnél ez utóbbi egyenlet helyettesíti E polarizáció esetén a B egyenletet, a többi (C,D,E) változatlan. Az utolsó egyenlet térkomponensekre vonatkozó indexek nélküli alakja, illetve a C, D, E egyenletek amelyekben ϕ helyett E_x szerepel:

$$E_x(m,n+1) = E_x(m,n) + \frac{\partial E_x}{\partial y} h_E + \frac{1}{2} h_E{}^2 \left(\frac{\partial^2 E_x}{\partial y^2} + \left(k_J{}^2 - k_K{}^2\right)E_x\right) \qquad \text{(B*)}$$

$$E_x(m-1,n) = E_x(m,n) - \frac{\partial E_x}{\partial z} h_N + \frac{1}{2} \frac{\partial^2 E_x}{\partial z^2} h_N{}^2 \qquad \text{(C*)}$$

$$E_x(m,n-1) = E_x(m,n) - \frac{\partial E_x}{\partial y} h_W + \frac{1}{2} \frac{\partial^2 E_x}{\partial y^2} h_W{}^2 \qquad \text{(D*)}$$

$$E_x(m+1,n) = E_x(m,n) + \frac{\partial E_x}{\partial z} h_S + \frac{1}{2} \frac{\partial^2 E_x}{\partial z^2} h_S{}^2 \qquad \text{(E*)}$$

B* egyenletet h_W-vel, D^* egyenletet h_E-vel szorozzuk a korábbiakhoz hasonlóan azért, hogy a megszorzott egyenletek összeadása során az elsőrendű deriváltak kiessenek. Hasonlóan C^* egyenletet h_S-sel, míg E^* egyenletet h_N-nel szorozzuk meg majd képezzük összegüket.

Az első összegegyenletben a csapásirányú elektromos térkomponens dőlés-, míg a másodikban a vertikális irány szerinti második deriváltja szerepel. Ezen második deriváltakat az egyenletekből kifejezve, majd behelyettesítve a Helmholtz egyenletbe E_x -re egy (F) alakú egyenletet kapunk, melyben az együtthatók a homogén esethez képest csak az A (m,n) pontbeli csapásirányú elektromos térkomponens együtthatójában mutat eltérést.

$$a^* = \left(h_W k_J^2 + h_E k_K^2\right)/\left(h_E + h_W\right) - 2\left(1/h_E h_W + 1/h_S h_N\right) \qquad \text{(G*)}$$

$$b^* = b \qquad c^* = c \qquad d^* = d \qquad e^* = e \qquad \text{(H*)}$$

Teljesen hasonló gondolatmenet alkalmazásával érhető el, hogy hasonló modellre (vertikális határfelület választ el két különböző hullámszámú térrészt) a megfelelő H polarizációs véges differencia egyenletet felírjuk. Ekkor a csapásirányú mágneses és a vertikális elektromos térkomponens folytonosságát kell érvényesíteni a vertikális határfelület mentén. A véges differencia egyenlet együtthatóinak levezetési módja itt is teljesen hasonló, mint E polarizációnál, és mivel a belső határfeltételek a két polarizációnál más komponensekre érvényesek, az együtthatók is eltérőek lesznek az E és H polarizációs esetben. Megállapítható, hogy ily módon az eltérő konduktivitású térrészeket elválasztó határfelületi (amely vízszintes vagy függőleges lehet) belső rácspontra a két polarizációra megadhatók. A fenti gondolatmenet folytatható akkor is, ha a véges differencia egyenletet olyan belső rácspontra írjuk fel, mely három vagy négy különböző konduktivitású derékszögű négyszög cella közös csúcspontja. A legáltalánosabb eset az, amikor négy eltérő konduktivitású cellának van egy közös pontja. Ekkor pl. E polarizációt feltételezve a vertikális határfelület mentén E_x és H_z, a vízszintes határfelület mentén pedig E_x és H_y folytonosságát kell előírni. Valamennyi belső határfeltétel figyelembevétele túlhatározott egyenletrendszerre vezet. Ezt úgy lehet elkerülni, hogy az eltérő konduktivitású cellák középpontjai között a cellákra jellemző konduktivitás értékek közötti lineáris változást tételezünk fel.

A következőkben a két polarizációra vonatkozó peremfeltételekkel foglalkozunk. Azt vizsgáljuk, hogy a modellezés során használt oldalirányú, felső és alsó rácslehatárolások során milyen feltételezésekből indulunk ki, és milyen közelítéseket fogadunk el peremfeltételekként.

A **H polarizációs peremfeltételek**. Először vizsgáljuk meg, hogy kell-e a levegőre rácsot fektetni? Az első Maxwell egyenletből –mivel $rot\vec{H} = \sigma\vec{E}$ - következik, hogy $rot\vec{H} = \vec{0}$, azaz levegőben a mágneses tér állandó, hisz egyetlen irány szerint sem változik. Tehát a csapásirányú mágneses térkomponens is állandó a levegőben (egyszerűség kedvéért feltételezzük, hogy a felszínen a csapásirányú mágneses térkomponens valós része 1.0, míg képzetes része 0.) . Mivel H polarizációnál a véges differencia egyenletrendszerben az ismeretlenek a rácspontbeli csapásirányú mágneses térkomponens érték, ezért a levegőben nem kell rácsot felvenni. A véges különbséges modellezésnél oldalirányban is le kell határolni a rácsot. Akkor járunk el helyesen, ha az inhomogenitástól olyan távol történik meg ez a lehatárolás, ahol a mágneses térnek már nincs dőlésirányú változása, azaz feltételezzük, hogy a felszín alatt, elegendően távol:

$$\frac{\partial H_x}{\partial y} = 0$$

Ha a dőlés szerinti második deriváltra is hasonló feltételezéssel élünk, akkor a H polarizációra felírt Helmholtz egyenletből

$$\frac{\partial^2 H_x}{\partial z^2} = i\varpi\mu\sigma H_x = -k^2 H_x$$

Ez az egyenlet a homogén féltérre vonatkozó megoldást adja, melyhez hozzávéve az idő szerinti harmonikus változást a csapásirányú mágneses térkomponens modell szélén lévő vertikális határ menti mélység- és időszerinti változása adható meg:

$$H_x(z,t) = H_{x0}e^{-kz}e^{+i\varpi t} = H_{x0}e^{-\sqrt{-i\varpi\mu\sigma}z}e^{+i\varpi t} = H_{x0}e^{-\frac{1-i}{\sqrt{2}}\sqrt{\varpi\mu\sigma}z}e^{+i\varpi t} = H_{x0}e^{-\sqrt{\frac{\varpi\mu\sigma}{2}}z}e^{\sqrt{\frac{\varpi\mu\sigma}{2}}iz}e^{+i\varpi t}$$

A formula alapján a felszíni érték -H_{x0}- ismeretében a felszíntől számított tetszőleges mélységben -ahol a rácshoz tartozóan sort veszünk fel- a térkomponens meghatározható (időfüggésre természetesen nincs szükség). Ha az inhomogenitás(oka)t magába foglaló tér rétegzett, akkor a modell szélén lévő, legszélső bal- és jobboldali oszlop rácspontjaiba a rétegzett féltérre jellemző értékeket állítjuk be.

Az **E polarizációs peremfeltételek**et először a levegőre adjuk meg és szorítkozzunk a rács függőleges széleire. Itt is érvényes, hogy $rot\vec{H} = \sigma\vec{E} = \vec{0}$, amiből következik:

$$\left(\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z}\right) = \sigma E_x = 0$$

Ezen esetben is feltételezhető, hogy az inhomogenitástól távol a H_z komponens dőlésirányban nem változik, tehát

$$\frac{\partial H_y}{\partial z} = 0 \Rightarrow H_y(z) = H_{y0}$$

Tehát H_y a levegőben a rács szélén z függvényében állandó és értéke a dőlésirányú felszíni mágneses térkomponenssel egyezik meg. Mivel az E polarizációs feladatnál a csapásirányú elektromos térkomponens meghatározása a cél, így az alábbi egyenletből levegőre határozzuk meg ezt a komponenst!

$$rot\vec{E} = -\frac{\partial \vec{B}}{\partial t} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x & 0 & 0 \end{vmatrix} = -i\varpi\mu\left(\vec{i}\,0 + \vec{j}H_y + \vec{k}H_z\right)$$

Amiből $\quad \frac{\partial E_x}{\partial z} = -i\varpi\mu H_y \Rightarrow E_x = -i\varpi\mu H_{y0} z$

tehát a rács szélein vertikálisan a csapásirányú elektromos térnek lineáris változását kell feltételezni. Emiatt viszont a levegőre is, ellentétben a másik polarizációval, rácsot kell fektetni. A felszín alatt a két szélső oszlop mentén lévő rácspontokban teljesen hasonló elvet érvényesítünk mint a másik polarizáció esetén.

A **lineáris egyenletrendszer megoldása**

A parciális differenciálegyenletnek megfelelő véges differenciaegyenlet attól függően, hogy rácsszegélyen, vagy a rács egy belső rácspontjára lett felírva más és más alakú. Ha valamennyi véges differenciaegyenlet ismert, akkor sor- vagy oszlop folytonosan adjuk meg őket. Ennek eredményeként az együtthatómátrix sáv struktúrájú, amit az egyenletrendszer megoldása során figyelembe lehet venni, függetlenül attól, hogy az egyenletrendszer direkt vagy iteratív módját válasszuk A következőkben az egyenletrendszer iteratív megoldásának egy lehetséges változatát mutatjuk be.

Az ötpontos rácsmódszer alkalmazásakor az 1. ábra jelöléseit használva a (F) alakú egyenlet írható fel. Balról jobbra sorfolytonosan felülről lefelé haladva a rácson, jelöljük a központi –m sor- és n oszlopszámmal jellemezhető- A pontot 0-val, míg a haladási iránynak megfelelően a többi rácspontnak C, D,B,E sorrendben feleljen meg az 1,2,3,4 jelölés. Ekkor a középpontbeli függvényérték a szomszédos négy rácspontbeli függvényérték lineáris kombinációjaként adható meg (F) egyenletből átrendezés után:

$$\varphi_0 = C\varphi_1 + D\varphi_2 + B\varphi_3 + E\varphi_4$$

A Gauss-Seidel iterációs eljárásnál a k-ik iterációban amikor az A pontra számítjuk ki a függvényértéket és feltételezve azt, hogy a minden egyes iterációnál a fenti sorfolytonos irány szerint haladunk írható (mivel a C és D pontokra a k-ik iterációban már számoltunk függvényértéket, míg a B és E pontokra még csak az ezt megelőző iterációs lépésben):

$$\varphi_{0,G-S}^{k} = C\varphi_1^{k} + D\varphi_2^{k} + B\varphi_3^{k-1} + E\varphi_4^{k-1}$$

Megjegyezzük, hogy az eljárás feltételezi, hogy már az első iteráció előtt valamennyi rácspontban van valamilyen (a peremfeltételek miatt rögzített vagy másutt valamilyen közelítő) megoldás. A Gauss-Seidel eljárás bizonyítottan konvergens, azonban a gyorsabb számítási végett gyakran végzünk szukcesszív túlrelaxációt (succesive over relaxation kezdőbetűiből rövidítve SOR), amely a k-ik lépésben a Gauss-Seidel eljárással meghatározott értéket egy konvergencia gyorsító faktorral súlyozza, de figyelembe veszi az ezt megelőző iterációs lépésben meghatározott értéket is:

$$\varphi_{0,SOR}^{k} = \tau\varphi_{0,G-S}^{k} + (1+\tau)\varphi_{0,SOR}^{k-1}$$

Ezen formulában a konvergencia gyorsító faktor (τ) csak szimmetrikus együtthatómátrix mellett határozható meg analitikusan (a Gauss-Seidel eljárás iterációs mátrixának spektrális sugarából, ami normából számítható), míg esetünkben az együtthatómátrix nem szimmetrikus volta miatt értékére csak becslés adható. A becslés a konvergencia gyorsító faktor valós részére vonatkozik, a képzetes résznek stabilizáló szerepe van.

Pontszerű elektromos dipólus forrás felszíni EM terének meghatározása 2D-s esetben (FEM 2.5D modellezés)

Azt a feladatot nevezzük 2.5D-snek, ahol 2D-s szerkezetet feltételezve az elektromos vagy mágneses gerjesztés pontszerűnek tekinthető. Ebben az esetben a pontforrás tere 3D-s.
Ekkor a megoldásnak eleget kell tennie a forrás tagokkal bővített Maxwell – egyenleteknek. Háromdimenziós források - elektromos és mágneses dipólusok – 2D-s szerkezetek fölötti terének meghatározására a Maxwell - egyenletek a következő alakúak [Takács (1981)]:
Ha az időszerinti függés $e^{i\varpi t}$ alakú, akkor:

$$rot\vec{E} = -i\varpi\mu\left(\vec{H} + \vec{M}_S\right)$$

$$rot\vec{H} = (\sigma + i\varpi\varepsilon)\vec{E} + \vec{j}_S$$

ahol $\vec{M}_S$-a térrészben jelenlevő mágneses dipólus forrás momentuma, $\vec{j}_S$ az elektromos forrástól származó áramsűrűség. A fenti egyenletek az elektromágneses tér helytől való függését a frekvencia tartományban írják le. Legyen a szerkezeti csapás iránya változatlanul párhuzamos az x tengellyel és az EM tér tetszőleges összetevője F. Írja le F viselkedését a csapás mentén F(x). A Fourier transzformáció révén a tértartományra vonatkozó F(x) összefüggéshez a térbeli hullámszám tartományra vonatkozó $\tilde{F}(k_x)$ összefüggés adható meg.

A k_x – t térbeli hullámszámnak nevezzük, és az x vonal azaz a szerkezeti csapás mentén a 2π távolságra eső ciklusok számát jelenti. Emiatt használatos a térbeli frekvencia elnevezés is, a dimenzió m^{-1}. Az egyváltozós Fourier transzformációval megadható , hogy az eredeti F(x) térkomponens függvényben az egyes k_x térbeli frekvenciájú komponensek milyen amplitúdóval és fázissal vannak jelen. a Amennyiben csak csapás, illetve dőlés irányú elektromos dipólus forrás gerjesztést tételezünk fel, továbbá az eltolási áramot elhanyagolhatónak tekintjük a vezetési áramhoz képest, a komponensekre felírt Maxwell egyenletek az x szerinti Fourier transzformált térben adhatjuk meg. Ha a tetszőleges irányú elektromos vagy mágneses komponenst $F(x, y, z)$-vel jelöljük, mely függvényről feltételezzük, hogy abszolút integrálható, akkor Fourier transzformáltja:

$$\widetilde{F}(k_x, y, z) = \int_{-\infty}^{\infty} F(x, y, z) e^{-ik_x x} dx$$

Az elektromágneses térkomponensek csapásirányú deriváltjainak Fourier transzformáltja pedig a parciális integrálási szabály alkalmazásával adható meg:

$$\int_{-\infty}^{\infty} \frac{\partial F(x, y, z)}{\partial x} e^{-ik_x x} dx = \left[F(x, y, z) e^{-ik_x x} \right]_{-\infty}^{\infty} + ik_x \int_{-\infty}^{\infty} F(x, y, z) e^{-ik_x x} dx = ik_x \widetilde{F}(k_x, y, z)$$

Ha $x \to \infty$ vagy $x \to -\infty$ akkor valamennyi térkomponens az x=0 helyen lévő dipólus forrás esetén zérushoz tart, így a jobboldali kifejezés első tagja is ezért igaz, hogy egy függvény parciális deriváltjának Fourier transzformáltja az irány szerinti térbeli hullámszám és magának a függvény Fourier transzformáltja szorzatának képzetesegység-szerese. A komponensekre vonatkozó Maxwell egyenletek felírásakor feltételezve, hogy vízszintes síkban csak elektromos gerjesztő tér van, továbbá felhasználva, hogy bármely térkomponens x szerinti deriváltjának Fourier transzformáltja a térkomponens Fourier transzformáltjának (ik_x) – szerese írható, hogy:

$$\partial \widetilde{E}_z / \partial y - \partial \widetilde{E}_y / \partial z = -\, i\omega\mu \widetilde{H}_x \qquad \text{(TM1)}$$

$$\partial \widetilde{E}_x / \partial z - i\, k_x \widetilde{E}_z = -\, i\omega\mu \widetilde{H}_y \qquad \text{(TE2)}$$

$$i\, k_x\, \widetilde{E}_y - \partial \widetilde{E}_x / \partial y = -\, i\omega\mu \widetilde{H}_z \qquad \text{(TE3)}$$

$$\partial \widetilde{H}_z / \partial y - \partial \widetilde{H}_y / \partial z = \sigma\, \widetilde{E}_x + \widetilde{j}_{sx} \qquad \text{(TE1)}$$

$$\partial \widetilde{H}_x / \partial z - i\, k_x \widetilde{H}_z = \sigma\, \widetilde{E}_y + \widetilde{j}_{sy} \qquad \text{(TM2)}$$

$$ik_x \widetilde{H}_y - \partial \widetilde{H}_x / \partial y = \sigma\, \widetilde{E}_z \qquad \text{(TM3)}$$

Az MT 2D-s estnek megfelelő HPOL egyenletek itt a TM jelölésűek, míg az ottani EPOL egyenleteknek itt a TE jelű három egyenlet felel meg.

Forrásmentes térrészre $(j_{sx} = j_{sy} = 0)$ a fenti egyenletekből a többi térösszetevő kifejezhető $\widetilde{E}_x$ és $\widetilde{H}_x$ segítségével, ami leegyszerűsíti feladatunkat, mert így csak két skalár függő változóra kell megoldást keresnünk. Mivel ebben az esetben –ellentétben a MT-hoz képest- a források tere 3D-s mint látni fogjuk a $k_x \neq 0$ miatt az egyenletek nem lesznek egymástól függetlenek. Az MT módszernél használt komponens egyenlet csoportosítás azonban itt is elvégezhető. Törekedjünk arra, hogy az MT H polarizációhoz tartozó egyenletek megfelelői szerepeljenek az egyik csoportban tehát a $\widetilde{H}_x, \widetilde{E}_y, \widetilde{E}_z$ komponensek (azaz a HPOL&TM móduszhoz tartozó komponensek), míg az E polarizációhoz tartozó $\widetilde{E}_x, \widetilde{H}_y, \widetilde{H}_z$ komponensek a másikban (EPOL&TE módusz). Egyetlen ″tiszta″ a TM móduszhoz tartozó egyenlet van ez a TM1 egyenlet. A második TE2 és TM3 egyenletekből $\widetilde{H}_y$, míg a harmadik TE3 és TM2 $\widetilde{H}_z$ eliminációjával adható meg:

$$\frac{\partial \widetilde{E}_z}{\partial y} - \frac{\partial \widetilde{E}_y}{\partial z} = -i\omega\mu \widetilde{H}_x$$

$$(\sigma + \frac{k_x^2}{i\omega\mu})\widetilde{E}_z = -\frac{\partial \widetilde{H}_x}{\partial y} - \frac{k_x}{\omega\mu}\frac{\partial \widetilde{E}_x}{\partial z}$$

$$-(\sigma + \frac{k_x^2}{i\omega\mu})\widetilde{E}_y = -\frac{\partial \widetilde{H}_x}{\partial z} + \frac{k_x}{\omega\mu}\frac{\partial \widetilde{E}_x}{\partial y} + \widetilde{j}_{sy}$$

Az így megadott TM egyenletekben $\widetilde{H}_x, \widetilde{E}_y, \widetilde{E}_z$ komponensek mellett tehát $\widetilde{E}_x$ is szerepel. A TE móduszhoz tartozó egyenletek között a TE1 számú adott. A következő TE3 és TM2 egyenletekből most $\widetilde{E}_y$, míg a harmadik TE2 és TM3 egyenletekből $\widetilde{E}_z$ eliminációjával határozható meg:

$$\frac{\partial \widetilde{H}_z}{\partial y} - \frac{\partial \widetilde{H}_y}{\partial z} = \sigma \widetilde{E}_x + \widetilde{j}_{sx}$$

$$-\left(i\omega\mu+\frac{k_x^2}{\sigma}\right)\tilde{H}_z=-\frac{\partial\tilde{E}_x}{\partial y}+\frac{ik_x}{\sigma}\frac{\partial\tilde{H}_x}{\partial z}-\frac{ik_x}{\sigma}\tilde{j}_{sy}$$

$$(i\omega\mu+\frac{k_x^2}{\sigma})\tilde{H}_y=-\frac{\partial\tilde{E}_x}{\partial z}-\frac{ik_x}{\sigma}\frac{\partial\tilde{H}_x}{\partial y}$$

Itt viszont $\tilde{E}_x, \tilde{H}_y, \tilde{H}_z$ komponensek mellett $\tilde{H}_x$ is megjelenik. Annak érdekében, hogy az ismeretlenek száma a két móduszra tovább csökkenjen, mindkét polarizációnál hasonlóan járjunk el. A ″tiszta″ polarizációs egyenletek baloldali két deriváltját fejezzük ki az ugyanazon polarizációhoz tartozó másik két egyenletből és azokat helyettesítsük be a ″tiszta″ polarizációs egyenletbe.
A TM és TE móduszhoz tartozó egyenletekben a távvezeték egyenletekhez hasonlóan folytonos impedancia és admittancia értékeket definiálhatók a következő módon (Stoyer , 1976):

$\zeta^M=\sigma(1-\frac{k_x^2}{k^2})$ - TM impedancia

$\zeta^E=i\omega\mu(1-\frac{k_x^2}{k^2})$ - TE impedancia

$\upsilon^M=i\omega\mu$ - TM admittancia

$\upsilon^E=\sigma$ - TE admittancia

Itt a MT-hoz hasonlóan $k^2=-i\varpi\mu\sigma$.Az impedancia és admittancia értékeket helyettesítsük be a TM és TE móduszhoz tartozó egyenletekbe, és bevezetve a

$$\xi=(k_x^2-k^2)^{-1}$$

jelölést a TM móduszra a következő parciális differenciál egyenletet kapjuk:

$$-\frac{\partial}{\partial y}\left(\frac{1}{\zeta^M}\frac{\partial \widetilde{H}_x}{\partial y}\right)-\frac{\partial}{\partial z}\left(\frac{1}{\zeta^M}\frac{\partial \widetilde{H}_x}{\partial z}\right)-ik_x\frac{\partial \xi}{\partial y}\frac{\partial \widetilde{E}_x}{\partial z}+ik_x\frac{\partial \xi}{\partial z}\frac{\partial \widetilde{E}_x}{\partial y}+$$
$$+\upsilon^M\widetilde{H}_x=-\frac{\partial}{\partial z}\left(\frac{\widetilde{j}_{sy}}{\zeta^M}\right)$$

TE móduszra vonatkozóan a parciális differenciálegyenlet hasonlóan kapva meg, írható:

$$-\frac{\partial}{\partial y}\left(\frac{1}{\zeta^E}\frac{\partial \widetilde{E}_x}{\partial y}\right)-\frac{\partial}{\partial z}\left(\frac{1}{\zeta^E}\frac{\partial \widetilde{E}_x}{\partial z}\right)+ik_x\frac{\partial \xi}{\partial y}\frac{\partial \widetilde{H}_x}{\partial z}-ik_x\frac{\partial \xi}{\partial z}\frac{\partial \widetilde{H}_x}{\partial y}+\upsilon^E\widetilde{E}_x=$$
$$=-\widetilde{j}_{sx}+ik_x\frac{\partial}{\partial y}\left(\xi\,\widetilde{j}_{sy}\right)$$

Ez a két egyenlet csupán az $\widetilde{E}_x$ és $\widetilde{H}_x$ komponenseket tartalmazza, azonban a TM móduszhoz tartozó egyenletben így megjelenik a 2D-s MT – től eltérően a csapás irányú elektromos tér, illetve a TE módusz egyenletében a csapás irányú mágneses tér. Összefoglalva a síkhullámú esethez képesti különbséget megállapítható, hogy

- ezen feladat a Fourier transzformált térben oldható meg
- két módusz között csatolás van
- a feladatot leíró egyenletekben a forrásból eredő tagok is megjelennek.

A két forráspolarizáció vizsgálatakor vagy csapásirányú, vagy rá merőleges és vízszintes (y) irányú gerjesztést tételezünk fel. Előbbi a TE , utóbbi a TM módusznak felel meg. Az eddigi levezetésekben együtt kezeltük a két gerjesztést. Felírva a TE módusznak megfelelő parciális differenciál egyenletrendszert:

$$-\frac{\partial}{\partial y}\left(\frac{1}{\zeta^M}\frac{\partial \widetilde{H}_x}{\partial y}\right)-\frac{\partial}{\partial z}\left(\frac{1}{\zeta^M}\frac{\partial \widetilde{H}_x}{\partial z}\right)-ik_x\frac{\partial \xi}{\partial y}\frac{\partial \widetilde{E}_x}{\partial z}+ik_x\frac{\partial \xi}{\partial z}\frac{\partial \widetilde{E}_x}{\partial y}+\upsilon^M\widetilde{H}_x=0$$

$$-\frac{\partial}{\partial y}\left(\frac{1}{\zeta^E}\frac{\partial \widetilde{E}_x}{\partial y}\right)-\frac{\partial}{\partial z}\left(\frac{1}{\zeta^E}\frac{\partial \widetilde{E}_x}{\partial z}\right)+ik_x\frac{\partial \xi}{\partial y}\frac{\partial \widetilde{H}_x}{\partial z}-ik_x\frac{\partial \xi}{\partial z}\frac{\partial \widetilde{H}_x}{\partial y}+\upsilon^E\widetilde{E}_x=-\widetilde{j}_{sx}$$

Teljesen hasonlóan a csapásirányra merőleges gerjesztés mellett (TM módusz):

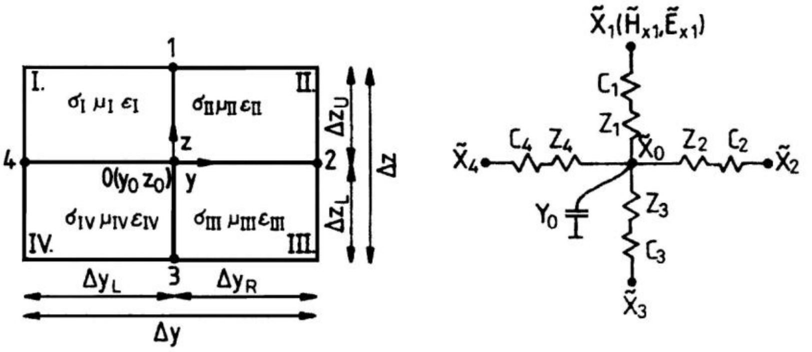
$$-\frac{\partial}{\partial y}\left(\frac{1}{\zeta^M}\frac{\partial \widetilde{H}_x}{\partial y}\right)-\frac{\partial}{\partial z}\left(\frac{1}{\zeta^M}\frac{\partial \widetilde{H}_x}{\partial z}\right)-ik_x\frac{\partial \xi}{\partial y}\frac{\partial \widetilde{E}_x}{\partial z}+ik_x\frac{\partial \xi}{\partial z}\frac{\partial \widetilde{E}_x}{\partial y}+\upsilon^M\widetilde{H}_x=-\frac{\partial}{\partial z}\left(\frac{\widetilde{j}_{sy}}{\zeta^M}\right)$$

$$-\frac{\partial}{\partial y}\left(\frac{1}{\zeta^E}\frac{\partial \widetilde{E}_x}{\partial y}\right)-\frac{\partial}{\partial z}\left(\frac{1}{\zeta^E}\frac{\partial \widetilde{E}_x}{\partial z}\right)+ik_x\frac{\partial \xi}{\partial y}\frac{\partial \widetilde{H}_x}{\partial z}-ik_x\frac{\partial \xi}{\partial z}\frac{\partial \widetilde{H}_x}{\partial y}+\upsilon^E\widetilde{E}_x=ik_x\frac{\partial}{\partial y}\left(\xi\,\widetilde{j}_{sy}\right)$$

Az egyenletrendszer megoldása során minden egyes rácspontra a Fourier transzformált csapás irányú elektromos és csapás irányú mágneses komponenst ($\widetilde{E}_x$ és $\widetilde{H}_x$) határozzuk meg.

Véges különbséges megoldás

A bemutatni kívánt módszer egy ötpontos rácsmódszer, azaz a középső rácspontbeli függvényértéket a négy szomszédos rácspontbeli függvényérték líneáris kombinációjaként adjuk meg. Swift majd Stoyer alkalmazta más jellegű gerjesztésekre ezt a távvezeték(felület) analógiát elsők között, melynek az a lényege, hogy a modellezni kívánt térrészre (derékszögű négyszög) hálót fektetve valamennyi , rácselemenként folytonos paraméterrel jellemezhető rácsrészlet diszkrét áramköri elemekből felépített áramkörrel helyettesíthető (3.ábra).



3.ábra

Így a középponti és a szomszédos rácspontok között diszkrét impedanciák (Z_i), csatolótagok (C_i), míg a középponti rácspont és a föld között diszkrét admittanciák (Y_0) adhatók meg. A diszkrét impedanciák és admittanciák a cellaelemenként állandó fizikai paraméterek és a rács geometriai jellemzőinek függvényei, míg a csatolótag független a rács geometriától (a két módusz közötti csatolás annál nagyobb, minél inkább eltávolodunk a modellezés során a távoli zónától). A távvezeték felület analógiát felhasználva csapás irányú gerjesztést feltételezve az alábbi véges differencia egyenletrendszer adható meg az ábrán látható rácsrészletre:

$$\sum_{i=1}^{4}\left(\frac{\tilde{H}_{ix}-\tilde{H}_{0x}}{-Z_i^M}+\frac{\tilde{E}_{ix}-\tilde{E}_{0x}}{C_i}\right)+Y_0^M\tilde{H}_{0x}=0$$

$$\sum_{i=1}^{4}\left(\frac{\tilde{E}_{ix}-\tilde{E}_{0x}}{-Z_i^E}+\frac{\tilde{H}_{ix}-\tilde{H}_{0x}}{C_i}\right)+Y_0^E\tilde{E}_{0x}=-\tilde{j}_{s0x}\Delta y\Delta x$$

Ebben az egyenletrendszerben a TM móduszra definiálható diszkrét impedancia (az 1-es és centrális rácspont között) és diszkrét admittancia, továbbá a módusztól független csatolótag a következő alakú:

$$\frac{1}{Z_1^M}=\frac{2}{\Delta z_U}\left(\frac{\Delta y_R}{\zeta_{II}^M}+\frac{\Delta y_L}{\zeta_I^M}\right)$$

$$Y_0^M=\Delta y_L\Delta z_U\upsilon_1^M+\Delta y_R\Delta z_U\upsilon_{II}^M+\Delta y_R\Delta z_L\upsilon_{III}^M+\Delta y_L\Delta z_L\upsilon_{IV}^M$$

$$\frac{1}{C_1}=2jk_x(\xi_I-\xi_{II})$$

Ha a gerjesztő dipólus forrás merőleges a szerkezeti csapás irányra, akkor teljesen hasonló véges differencia egyenletrendszert lehet megadni, az eltérés a jobb oldalon az inhomogén tag vonatkozásában van csupán. A forrástagokat elosztott paraméterként kezeljük csapás irányban kiterjedés nélkül. A csapás irányú forrástagot két (egymással érintkező) negyed cellában elkenten képzeljük el, míg a dőlés irányú forrástagot egyetlen cellában elkenve. Ez utóbbit az indokolja, hogy a forrástag y- és z szerinti deriváltját kell venni.

A véges differencia egyenleteket oszlop folytonosan balról jobbra haladva egy olyan egyenletrendszert kapunk, melynek $[Q]$ együtthatómátrixa nem szimmetrikus, *4mn*4mn* elemet tartalmaz és blokk tridiagonális struktúrája van. Az alkalmazott rácsokra általában az jellemző, hogy több oszlopuk (*n*) mint soruk (*m*) van. Az ilyen típusú egyenletrendszerek egyik lehetséges megoldási módja a blokk tridiagonális LU dekompozíció, melynek lépéseit a következőkben adjuk meg.

$$[Q]\vec{X}=\{[F]_l[M]_l[N]_l\}_1^n\vec{X}=[L][U]\vec{X}=\{[C]_l[I]_l[0]\}_1^n\{[0][A]_l[N]_l\}_1^n\vec{X}=\vec{S} \qquad \text{LU1}$$

Itt valamennyi (összesen n) blokk *4m*4m*-es méretű mátrix.
Az LU dekompozícióknál szokásos $[U]\vec{X} = \vec{Y}$ egyenletet bevezetve az első lépés az $[L]\vec{Y} = \vec{S}$ megoldása, mely rekurzív módon adható meg:

$$\vec{Y}_1 = \vec{S}_1 \qquad \vec{Y}_l = \vec{S}_l - [F]_l [A]_{l-1}^{-1} \vec{Y}_{l-1} \qquad l = 2,3,\ldots\ldots n \qquad \text{LU2}$$

Ha más típusú forrásunk van, vagy más helyen van a forrás - $\vec{R}$ - a megoldást az előzően teljesen lehet megadni, azaz:

$$\vec{Y}_1 = \vec{R}_1 \qquad \vec{Y}_l = \vec{R}_l - [F]_l [A]_{l-1}^{-1} \vec{Y}_{l-1} \qquad l = 2,3,\ldots\ldots n \qquad \text{LU3}$$

Melyből a $[U]\vec{X} = \vec{Y}$ egyenlet (egyúttal az LU1) megoldása:

$$\vec{X}_n = [A]_n^{-1} \vec{Y}_n \qquad \vec{X}_l = [A]_l^{-1} \left(\vec{Y}_l - [N]_l \vec{X}_{l+1} \right) \qquad l = 2,3,\ldots\ldots n-1, n-2,\ldots.2,1 \qquad \text{LU4}$$

Az ismertetett megoldás tehát figyelembe veszi az együtthatómátrix blokkos sávstruktúráját, és a legnagyobb számításigényű művelet a *4m*4m*-es méretű $[A]_l$ blokkok invertálása (összesen n-szer) lesz. Ennek eredményét ugyanazon frekvencia és model esetén máshol elhelyezkedő vagy más típusú forrásra fel lehet használni.
A Fourier transzformált csapásirányú komponensek ismeretében a Fourier transzformált nem csapásirányú EM térkomponensek a két polarizációra megadott komponens egyenletekből számíthatók a forrásmentes térrészre. TE módusznál a pontszerű adódipólust magában foglaló (vagy a csapásirányú adódipólus felezőpontján átmenő) csapásirányra merőleges síkban E_x mellett H_y és H_z lesz zérustól különböző. E_x Fourier transzformáltját a megfelelő parciális diff. egyenletrendszer (j_{sx} az inhomogén tag) numerikus megoldása adja, míg a forrástag nélküli TE2 és TM3 egyenletekből H_y, míg TE3 és TM2 egyenletekből H_z Fourier transzormáltja határozható meg:

$$\tilde{H}_y = \frac{-\dfrac{\partial \tilde{E}_x}{\partial z}}{\left(1 - k_x^2 / k^2\right) i \varpi \mu} - \frac{i k_x}{k_x^2 - k^2} \frac{\partial \tilde{H}_x}{\partial y}$$

$$\tilde{H}_z = \frac{\dfrac{\partial \tilde{E}_x}{\partial y}}{\left(1 - k_x^2 / k^2\right) i \varpi \mu} - \frac{i k_x}{k_x^2 - k^2} \frac{\partial \tilde{H}_x}{\partial z}$$

Szembetűnő a formai hasonlóság, az eltérés a parciális deriváltakban, ill. a jobb oldalon lévő első tag előjelében van.
TM módusz esetén az adó dipólust magába foglaló síkban E_y mellett H_x és E_z lesz zérustól eltérő. A csapásirányú mágneses térkomponens Fourier transzformáltja a csapásirányra merőleges elektromos dipólus gerjesztést feltételező diff. egyenletrendszer megoldása során adható meg. A TE3 és TM2 egyenletekből E_y, míg a TE2 és TM3 egyenletekből E_z Fourier transzformáltját lehet kifejezni.

$$\tilde{E}_y = \frac{\frac{\partial \tilde{H}_x}{\partial z}}{\left(1 - k_x^2 / k^2\right)\sigma} - \frac{ik_x}{k_x^2 - k^2}\frac{\partial \tilde{E}_x}{\partial y}$$

$$\tilde{E}_z = \frac{-\frac{\partial \tilde{H}_x}{\partial z}}{\left(1 - k_x^2 / k^2\right)\sigma} - \frac{ik_x}{k_x^2 - k^2}\frac{\partial \tilde{E}_x}{\partial z}$$

Annak érdekében, hogy a tértartománybeli értékeket megkapjuk, a hullámszám tartományból a tértartományba váló visszatéréshez inverz Fourier transzformációt használunk:

$$F(x, y, z) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \tilde{F}(k_x, y, z) e^{ik_x x} dk_x$$

Itt a jelölés ugyanaz, mint a Fourier transzformáció definíciójánál. Ha a modellezést az adódipólust magába foglaló csapásirányra merőlegesen végezzük el, azaz az x=0 síkra, akkor a transzformáció egyszerűsödik.

Mivel mind a három zónában az EM tér elektromos és mágneses térkomponensek gyengülése r^{-3} és/vagy r^{-2} – val arányos ezért az impedancia (Z) abszolút értékére vonatkozó kifejezésben a távolságfüggés részben kiesik és azt várjuk, hogy az inhomogén szerkezetek sokkal jobban leképeződnek, mint csupán az EM térerősség komponensek használatakor.
A TE impedancia abszolút értéke:

$$|Z_{TE}| = \frac{|E_x|}{|H_y|} = \frac{\left| \int_0^{k_{x\max}} \widetilde{E}_x dk_x \right|}{\left| \int_0^{k_{x\max}} \left[\frac{\frac{\partial \widetilde{E}_x}{\partial z}}{(k_x^2/k^2 - 1) i\omega\mu} - \frac{ik_x}{k_x^2 - k^2} \frac{\partial \widetilde{H}_x}{\partial y} \right] dk_x \right|}$$

TM impedancia abszolút értéke:

$$|Z_{TM}| = \frac{|E_y|}{|H_x|} = \frac{\left| \int_0^{k_{x\max}} \left[\frac{\frac{\partial \widetilde{H}_x}{\partial z}}{(1 - k_x^2/k^2)\sigma} - \frac{ik_x}{k_x^2 - k^2} \frac{\partial \widetilde{E}_x}{\partial y} \right] dk_x \right|}{\left| \int_0^{k_{x\max}} \widetilde{H}_x dk_x \right|}$$

Ezek a formulák azért hasznosak, mert az impedancia amplitúdó távolság függése korántsem olyan mértékű, mint az egyes komponensek távolság szerinti csillapodása. Az impedancia fázisát az MT 2D esetnél megadott módon számítjuk. Az impedancia fázisa pedig ugyanazt az információt hordozza a távoli zónában a dipólus gerjesztésnél, mint az MT 2D-s fázis érték, ugyanakkor az átmeneti zónában is van frekvenciális függés, ellentétben a közeli zónával.

A szigorúan 2D –s inhomogenitásokat feltételezve a MT –nél a H_z komponenst a csapásirányú elektromos tér dőlés irányú változása hozza létre. Itt az összefüggés összetettebb. A dipólus elektromágneses terét első lépésben a Fourier transzformált térben számítjuk. A tértartománybeli H_z térkomponens inverz Fourier – transzformációval adható meg, a következő módon [Pethő (1995)]:

$$H_z(x,y,z) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \left[\frac{\frac{\partial \widetilde{E}_x}{\partial y}}{(1 - k_x^2/k^2) i\omega\mu} - \frac{ik_x}{k_x^2 - k^2} \frac{\partial \widetilde{H}_x}{\partial z} \right] e^{ik_x x} dk_x$$

Ez az inverz transzformáció az adódipólust magába foglaló x = 0 síkban egyszerűsödik. Így a H_z (0,y,z) komponensre írható:

$$H_z(0,y,z) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\left[\frac{\dfrac{\partial \tilde{E}_x}{\partial y}}{(1-k_x^2/k^2)i\omega\mu} - \frac{ik_x}{k_x^2-k^2}\frac{\partial \tilde{H}_x}{\partial z}\right]dk_x$$

A dőlésirányú elektromos dipólusforrás esetén a H_z (0,y,z) komponens 2D –s inhomogenitásokra zérus lesz a gerjesztő elektromos dipólus forrást magába foglaló vertikális síkban. A fenti formula alapján állítható, hogy minél távolabb van a megfigyelési hely a csapásirányú elektromos dipólus forrástól, vagy adott adó – vevő távolság mellett minél nagyobb frekvencián mérünk, annál inkább a 2D-s MT –nél megismert összefüggés válik dominánssá, ugyanis az integrandusz szögletes zárójelében lévő második tag elhanyagolhatóvá válik. Jelen esetben mivel az átmeneti zónában is modellezünk a csapásirányú mágneses tér vertikális irány szerinti változása is befolyásoló tényező. Annak érdekében, hogy az inhomogenitás hatását kiemeljük itt (TE módusz) célszerű a normált H_z amplitúdó eloszlást megadni. H_z fázisa - mely a korábbiakhoz hasonlóan a képzetes és valós rész hányadosának arctan-se – függ az adódipólustól való távolságtól, ezért itt is célszerű elvégezni a homogén esetre történő normálást. Ez itt különbségi fázis értékek megadását jelenti.

Numerikus modellezési példákat és a modellezés során szerzett tapasztalokat a segédlet következő részében mutatunk be.

FORMALISM COMPARISON OF 2-D MT AND 2.5-D FEM USING ELECTRIC DIPOLE SOURCE

INTRODUCTION

The two-dimensional (2-D) problem emerges when the geological structure can be characterized as an elongated one. The axis of elongation is called as the structural strike direction. Special attention has made to this situation because the real three-dimensional situation can frequently be approximated by elongated structure. Besides the geometry the conductivity is the physical parameter which does not show any variation along the strike. Two types of electromagnetic (EM) source fields are assumed: plane wave EM field which is characteristic of magnetotellurics (MT) and radiomagnetotellurics (RMT) including VLF method utilizing the very low frequency EM fields of distant transmitters. In the second case the source is finite, it is a grounded wire. A 2-D model with a 3-D source is called as 2.5-D case. In both situations we assume $e^{i\varpi t}$ time dependent EM field variation and x coincides with the strike direction. The basic relationships governing the electromagnetic phenomenon are the Maxwell's equations. There is no difference in the form of the first Maxwell's equation, i.e.:

$$rot\vec{E} = -\partial\vec{B}/\partial t \tag{1}$$

The second Maxwell's equation for MT: is:

$$rot\vec{H} = \vec{j} \tag{2}$$

Taking the effect of source term into account:

$$rot\vec{H} = \vec{j} + \vec{j}_s \tag{3}$$

Where $\vec{j}_s = Id\vec{s}\,\delta(\vec{r})$ making possible a point source approximation.

BASIC 2-D MT FORMALISM

In case of MT if the partial derivative with respect x equals zero and the component equations of (1) are as follows:

$$\left(\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z}\right) = -i\varpi\mu H_x \tag{4}$$

$$\frac{\partial E_x}{\partial z} = -i\varpi\mu H_y \quad (5)$$

$$\frac{\partial E_x}{\partial y} = i\varpi\mu H_z \quad (6)$$

Similarly from equation (2) in 2-D MT :

$$\left(\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z}\right) = \sigma E_x \quad (7)$$

$$\frac{\partial H_x}{\partial z} = \sigma E_y \quad (8)$$

$$-\frac{\partial H_x}{\partial y} = \sigma E_z \quad (9)$$

If equations (5), (6), (7) are taken into one set of equations then only E_x, H_y, H_z components, and similarly in (4), (8), (9) H_x, E_y, E_z can be found. The conclusion is that independently of the angle of incident the plane wave the EM field can be divided into two parts, which can be mathematically treated separately. The first mode is called TE mode, the second is the TM mode. In order to determine the 2-D MT equations for homogeneous case in the first group the value of magnetic field components from (5), (6) into (7) and in the second group the value of electric field components from (8), (9) into (4) can be substituted. In this way the most comprehensive form for the TE mode is:

$$\frac{\partial^2 E_x}{\partial y^2} + \frac{\partial^2 E_x}{\partial z^2} = i\varpi\mu\sigma E_x \quad (10)$$

Similarly for TM mode:

$$\frac{\partial^2 H_x}{\partial y^2} + \frac{\partial^2 H_x}{\partial z^2} = i\varpi\mu\sigma H_x \quad (11)$$

BASIC 2.5-D FORMALISM

The original three-dimensional problem can be reduced to a series of two - dimensional ones, if the Maxwell's equations are Fourier transformed into the strike directional wave number (k_x) domain [1]. We apply the following Fourier transform pair:

$$\tilde{F}(k_x, y, z) = \int_{-\infty}^{\infty} F(x, y, z) e^{-ik_x x} dx \quad (12)$$

$$F(x, y, z) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \tilde{F}(k_x, y, z) e^{ik_x x} dk_x \quad (13)$$

The Fourier transform of the partial derivative of the EM field components with respect *x* is also needed which is proportional the Fourier transform of the component itself:

$$\int_{-\infty}^{\infty}\frac{\partial F(x,y,z)}{\partial x}e^{-ik_x x}dx=\left[F(x,y,z)e^{-ik_x x}\right]_{-\infty}^{\infty}+ik_x\int_{-\infty}^{\infty}F(x,y,z)e^{-ik_x x}dx=ik_x\tilde{F}(k_x,y,z) \qquad (14)$$

For this situation the Fourier transform of Maxwell's components equations are as follows:

$$\partial\tilde{E}_z/\partial y-\partial\tilde{E}_y/\partial z=-\,i\omega\mu\,\tilde{H}_x \qquad (15)$$

$$\partial\tilde{E}_x/\partial z-i\,k_x\tilde{E}_z=-\,i\omega\mu\,\tilde{H}_y \qquad (16)$$

$$i\,k_x\,\tilde{E}_y-\partial\tilde{E}_x/\partial y=-\,i\omega\mu\,\tilde{H}_z \qquad (17)$$

$$\partial\tilde{H}_z/\partial y-\partial\tilde{H}_y/\partial z=\sigma\,\tilde{E}_x+\tilde{j}_{sx} \qquad (18)$$

$$\partial\tilde{H}_x/\partial z-i\,k_x\tilde{H}_z=\sigma\,\tilde{E}_y+\tilde{j}_{sy} \qquad (19)$$

$$ik_x\tilde{H}_y-\partial\tilde{H}_x/\partial y=\sigma\,\tilde{E}_z \qquad (20)$$

In this case the two modes can not be separated because the only poor TE component equation is (18) and that of TM mode is (15). Here TE mode denotes the situation when the direction of electric dipole source is parallel to the strike and TM mode corresponds to the case when it is perpendicular to it. However, instead of having three component it is more comfortable to work with two for each mode. Introducing the following notation as $k^2=-i\varpi\mu\sigma$, $\zeta^M=\sigma\,(1-\frac{k_x^2}{k^2})$, $\zeta^E=i\omega\mu\,(1-\frac{k_x^2}{k^2})$, $\upsilon^M=i\omega\mu$, $\upsilon^E=\sigma$ a partial differential equation system can be derived for the two polarizations. From equations (15)-(20) for TE mode[2]:

$$-\frac{\partial}{\partial y}\left(\frac{1}{\zeta^M}\frac{\partial\tilde{H}_x}{\partial y}\right)-\frac{\partial}{\partial z}\left(\frac{1}{\zeta^M}\frac{\partial\tilde{H}_x}{\partial z}\right)-ik_x\frac{\partial\xi}{\partial y}\frac{\partial\tilde{E}_x}{\partial z}+ik_x\frac{\partial\xi}{\partial z}\frac{\partial\tilde{E}_x}{\partial y}+\upsilon^M\tilde{H}_x=0 \qquad (21)$$

$$-\frac{\partial}{\partial y}\left(\frac{1}{\zeta^E}\frac{\partial\tilde{E}_x}{\partial y}\right)-\frac{\partial}{\partial z}\left(\frac{1}{\zeta^E}\frac{\partial\tilde{E}_x}{\partial z}\right)+ik_x\frac{\partial\xi}{\partial y}\frac{\partial\tilde{H}_x}{\partial z}-ik_x\frac{\partial\xi}{\partial z}\frac{\partial\tilde{H}_x}{\partial y}+\upsilon^E\tilde{E}_x=-\tilde{j}_{sx} \qquad (22)$$

Applying the same equations it can be derived for TM mode:

$$-\frac{\partial}{\partial y}\left(\frac{1}{\zeta^M}\frac{\partial\tilde{H}_x}{\partial y}\right)-\frac{\partial}{\partial z}\left(\frac{1}{\zeta^M}\frac{\partial\tilde{H}_x}{\partial z}\right)-ik_x\frac{\partial\xi}{\partial y}\frac{\partial\tilde{E}_x}{\partial z}+ik_x\frac{\partial\xi}{\partial z}\frac{\partial\tilde{E}_x}{\partial y}+\upsilon^M\tilde{H}_x=-\frac{\partial}{\partial z}\left(\frac{\tilde{j}_{sy}}{\zeta^M}\right) \qquad (23)$$

$$-\frac{\partial}{\partial y}\left(\frac{1}{\zeta^E}\frac{\partial \widetilde{E}_x}{\partial y}\right)-\frac{\partial}{\partial z}\left(\frac{1}{\zeta^E}\frac{\partial \widetilde{E}_x}{\partial z}\right)+ik_x\frac{\partial \xi}{\partial y}\frac{\partial \widetilde{H}_x}{\partial z}-ik_x\frac{\partial \xi}{\partial z}\frac{\partial \widetilde{H}_x}{\partial y}+\upsilon^E\widetilde{E}_x=ik_x\frac{\partial}{\partial y}\left(\xi\,\widetilde{j}_{sy}\right) \quad (24)$$

Depending upon the source term there are differences on the right-hand sides of the TE and TM mode equations. The similarity is that only strike directional EM field components are in the TE - (10) and (20), (21) - and TM mode - (11) and (23), (24) - partial differential equations. However, in plane wave situation the two modes separate, so one equation is enough to describe the problem in mathematical sense, opposite to the dipole source case when two equations in the strike directional wave number domain are needed. Having the solutions to these equations in (k_x) domain inverse Fourier transform (13) is needed to get solution in the space domain.

EM FIELD COMPONENTS WITH DIRECTION DIFFERENT FROM STRIKE AND IMPEDANCES FOR THE TWO EXCITATIONS IN THE TWO MODES

In case of plane wave TE mode this component can be expressed from equations (5), (6):

$$H_y=-\frac{1}{i\varpi\mu}\frac{\partial E_x}{\partial z} \qquad H_z=\frac{1}{i\varpi\mu}\frac{\partial E_x}{\partial y}$$

and for dipole excitation from equations (15)-(20) with the assumption of source free terms. Using inverse transform (13) for the vertical plane containing the source (x=0):

$$H_y(0,y,z)=\frac{1}{\pi}\int_0^\infty\left[\frac{-\dfrac{\partial \widetilde{E}_x}{\partial z}}{(1-k_x^2/k^2)i\omega\mu}-\frac{ik_x}{k_x^2-k^2}\frac{\partial \widetilde{H}_x}{\partial y}\right]dk_x \quad (25)$$

$$H_z(0,y,z)=\frac{1}{\pi}\int_0^\infty\left[\frac{\dfrac{\partial \widetilde{E}_x}{\partial y}}{(1-k_x^2/k^2)i\omega\mu}-\frac{ik_x}{k_x^2-k^2}\frac{\partial \widetilde{H}_x}{\partial z}\right]dk_x \quad (26)$$

Similarly for TM mode, plane wave situation from equations (8), (9):

$$E_y=\frac{1}{\sigma}\frac{\partial H_x}{\partial z} \qquad E_z=-\frac{1}{\sigma}\frac{\partial H_x}{\partial y}$$

For dipole excitation:

$$E_y(0,y,z)=\frac{1}{\pi}\int_0^\infty\left[\frac{\dfrac{\partial \widetilde{H}_x}{\partial z}}{(1-k_x^2/k^2)\sigma}-\frac{ik_x}{k_x^2-k^2}\frac{\partial \widetilde{E}_x}{\partial y}\right]dk_x \quad (27)$$

$$E_z(0,y,z)=\frac{1}{\pi}\int_0^{\infty}\left[\frac{-\frac{\partial \tilde{H}_x}{\partial y}}{(1-k_x^2/k^2)\sigma}-\frac{ik_x}{k_x^2-k^2}\frac{\partial \tilde{E}_x}{\partial z}\right]dk_x \qquad (28)$$

In the integrands the first term is dominant over the second one with decreasing k_x values. The case of $k_x=0$ corresponds to the plane wave situation. Approaching this in the vertical plane involving the source mainly the same directional variation of the same strike directional EM field component influences the behaviour of the vertical or the other horizontal (perpendicular to the strike, y) field components as in plane wave excitation case. At fixed frequency this situation can be approached by increasing transmitter-receiver distance which results in lower signal to noise ratio on the receiver site.

The basic relationship of MT -yields the resistivity over a half-space-includes the absolute value of impedance and it is the impedance frequency dependence which ensures the apparent resistivity frequency dependence. In the near-field zone the horizontal electric and the magnetic fields are independent of the frequency, in the far-field zone the EM fields decay at the same rate as $1/r^3$. The region between these two zones is called the transition zone in which the electric field varies as $1/r^3$ and the magnetic field decays at a rate between $1/r^2$ and $1/r^3$ and both depend upon frequency [3]. To get rid of or at least to decrease the effect of EM fields' decay with respect transmitter-receiver distance the use of impedances can be suggested even in transition zone neighbouring far-field zone. On the other hand it can result in the connection between MT and controlled source FEM apart from near-field zone. For TE mode at first plane wave case (using (5)):

$$|Z_{xy}|=\frac{|E_x|}{|H_y|}=\frac{\varpi\mu|E_x|}{\left|\frac{\partial E_x}{\partial z}\right|} \qquad (29)$$

For strike directional dipole source:

$$|Z_{TE}|=\frac{|E_x|}{|H_y|}=\frac{\left|\int_0^{k_{x\max}}\tilde{E}_x dk_x\right|}{\left|\int_0^{k_{x\max}}\left[\frac{\frac{\partial \tilde{E}_x}{\partial z}}{(k_x^2/k^2-1)i\omega\mu}-\frac{ik_x}{k_x^2-k^2}\frac{\partial \tilde{H}_x}{\partial y}\right]dk_x\right|} \qquad (30)$$

In the TM mode for plane wave:

$$|Z_{yx}|=\frac{|E_y|}{|H_x|}=\frac{\left|\frac{\partial H_x}{\partial z}\right|}{\sigma|H_x|} \qquad (31)$$

If the source is perpendicular to the strike then:

$$\left|Z_{TM}\right| = \frac{\left|E_y\right|}{\left|H_x\right|} = \frac{\left| \int\limits_0^{k_{x\max}} \left[\frac{\frac{\partial \widetilde{H}_x}{\partial z}}{(1-k_x^2/k^2)\sigma} - \frac{ik_x}{k_x^2-k^2}\frac{\partial \widetilde{E}_x}{\partial y} \right] dk_x \right|}{\left| \int\limits_0^{k_{x\max}} \widetilde{H}_x dk_x \right|} \qquad (32)$$

The vertical magnetic field determination has special significance in geological interpretation [4]. For example in RMT this is the EM component which can not be found in the primer EM field opposite to the horizontal electric dipole excitation. In this second situation the use of normalized vertical magnetic field is suggested to the homogeneous case. The use of impedance amplitude and phase is also recommended because on the basis of it current channelling and charge accumulation can be analysed for 2.5D case [5].

SUMMARY

In this paper emphasis was put on the comparison of the formalism of plane wave and electric dipole excitation for elongated conductivity structure. Only strike directional EM field components are in the TE and TM mode partial differential equations. The two modes separate in plane wave excitation and they are coupled in 2.5D FEM. After the partial differential equations the determination of EM field components different from strike direction and impedances were also presented. The formalism is more complicated for 2.5D FEM case, because there are more influencing factors comparing with 2D MT. From the formalism it is obvious that measurement and modelling data can be compared for the far-field zone.

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REFERENCES

[1] STOYER, C.H., GREENFIELD, R.J.: **Numerical solutions of the response of a two-dimensional earth to an oscillating magnetic dipole source**. Geophysics, 1976, Vol. 41. pp.519-530

[2] PETHŐ, G., FICSÓR, L.: **Source polarization effect in case of elongated surface inhomogeneities covering transition zone**. 63rd EAGE Conference & Exhibition, Extended Abstracts, 2001, Vol. 2, P050, ISBN 90-73781-18-3, Amsterdam
[3] ZONGE, K.L., HUGHES,L.J.: **Controlled source audio-frequency magnetotellurics**. In: Electromagnetic methods in applied geophysics, Vol.2. ed. M.N.Nabighian, 1987, SEG
[4] TAKÁCS, E.: **The role of the vertical pulsations of the magnetic field in magnetotelluric measurements**. Acta Geodetica, Geophysica et Montanistica Acad. Sci. Hung., 1971, Tom.6 (1-2), pp. 99-110
[5] PETHŐ, G., KAIKKONEN, P., VANYAN, L., L.: **Numerical modelling for the effect of a 2-D seafloor trench on sea-bottom EM measurements using horizontal electric dipole sources**. Geophysica, 1995, Vol. 31., pp. 1-21, Helsinki

COMPARISON OF 2-D VLF AND 2.5-D HED'S FAR FIELD REGIME EM FIELDS

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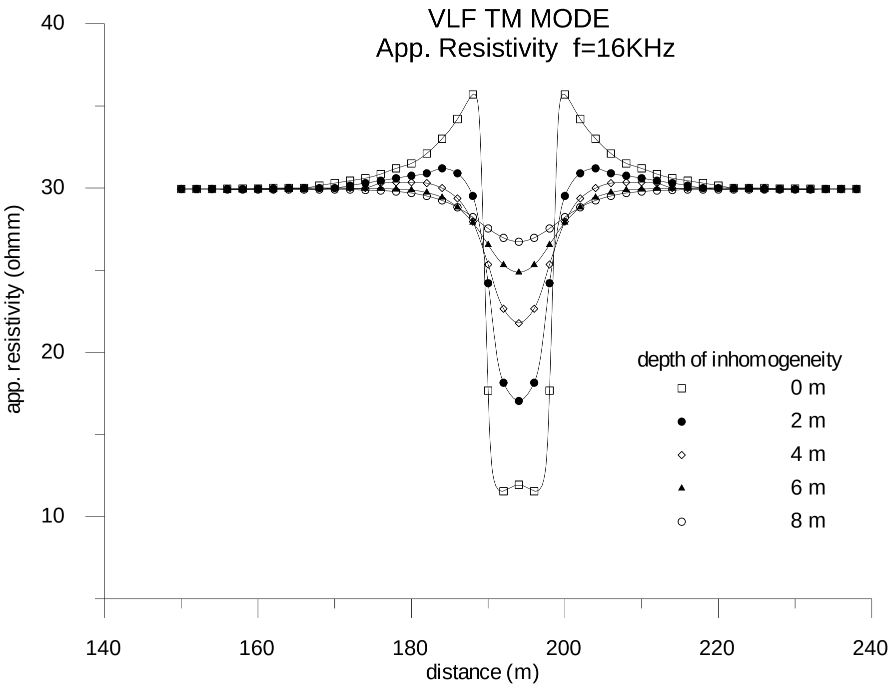
INTRODUCTION

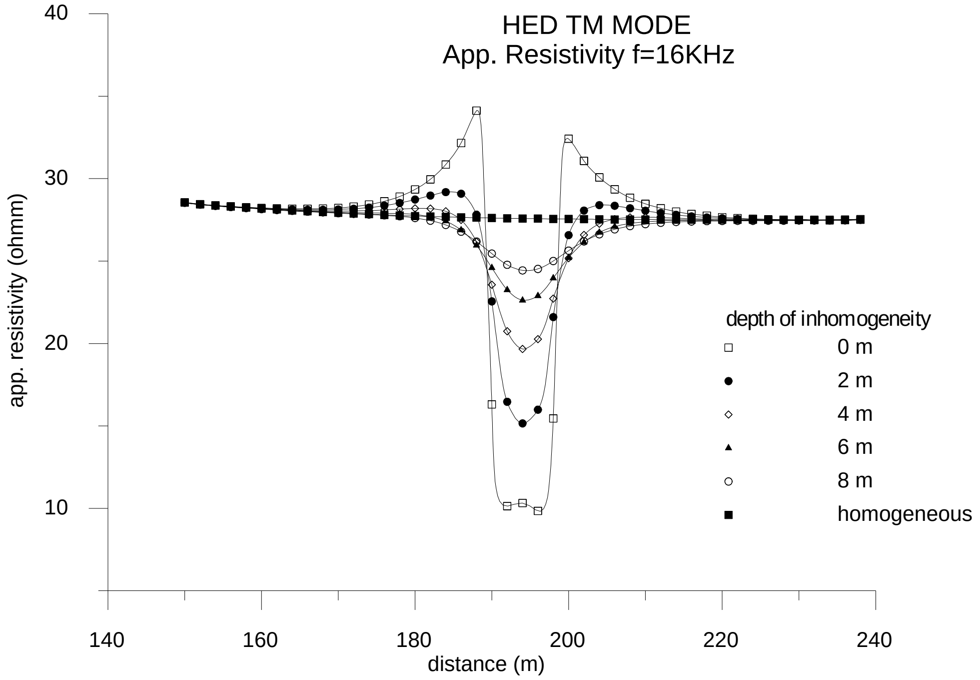
VLF method applies the very low frequency EM fields of distant transmitters in the frequency range of 10-30KHz. If the investigated conductivity structure is elongated along the structural strike and the source field is an EM plane wave, then the problem is a poor 2-D one. In the second case the EM source is a horizontal electric dipole source which is finite and it can be considered as a point source in the structural strike. A 2-D model excited by this 3-D source is called as 2.5-D situation. For these two situations the partial differential equations are derived and their solutions are also presented in [1]. In case of elongated conductivity structures special emphasis has to be put on the source polarization. Distinction is made between TE mode –when the electric source field is parallel to the strike- and TM mode, if the magnetic source field is parallel to it. Assuming horizontal electric dipole (HED) source field (just like in case of other dipole source with $e^{i\varpi t}$ time dependency) three zones can be separated [2] over the surface of the homogeneous half-space: far field when horizontal *E, H* decay as $1/r^3$ (*r* denotes the transmitter receiver distance) and the penetration depth depends on frequency and resistivity; transition zone in which the electric field varies as $1/r^3$ and the magnetic field decays as $1/r^2$ to $1/r^3$ and both depend upon frequency. The third zone is the near-field zone in which the horizontal *E* and *H* are independent of the frequency, *E* decays as $1/r^3$ and *H* as $1/r^2$. In the near-field zone the penetration depth can be controlled by the geometry opposite to the transition zone when besides the geometry the frequency is also an influencing factor. Numerical modelling results were presented in [3] [4] where the source polarization effect was investigated covering actually the whole transition zone. In this 2.5-D investigation the part of transition zone bordering the far field zone will be compared with the poor 2D MT case at 16 KHz frequency.

MODELLING IN THE TWO MODES FOR THE TWO EXCITATIONS

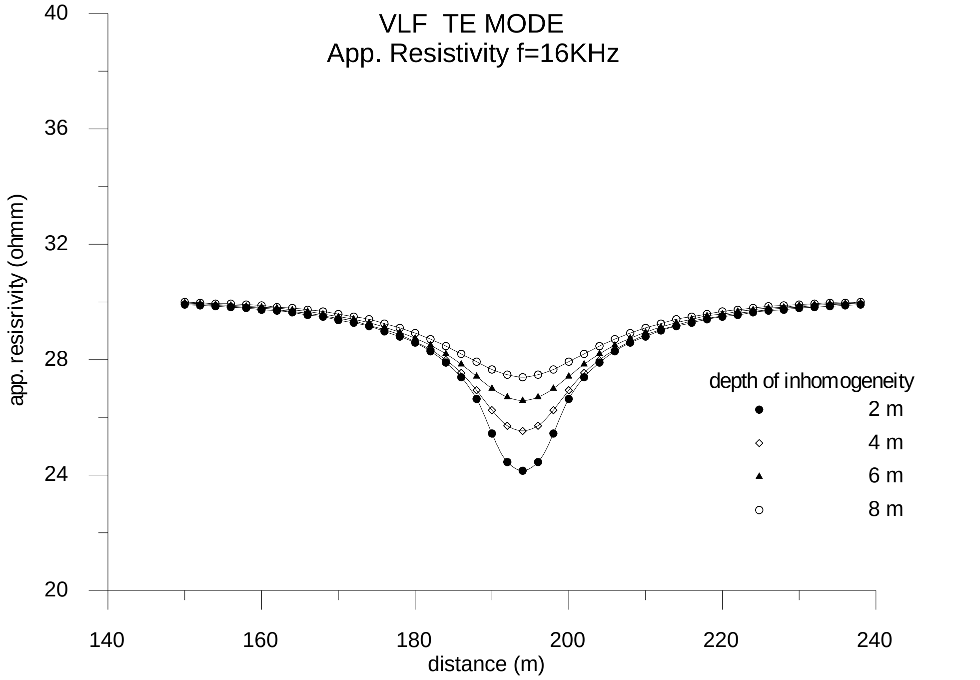
The inhomogeneity is an elongated prism with a square 8m*8m cross-section. Its resistivity is 15 ohmm and it is imbedded into a homogeneous half-space of 30ohmm. It is located at different depths: 0, 2, 4, 6, 8m and the axis of the prism is parallel to the surface. For the two kinds of excitations different grids were applied for the finite difference (FD) modelling of EM fields. However, in the vicinity of the inhomogeneity the same grid elements (2m both in horizontal and vertical



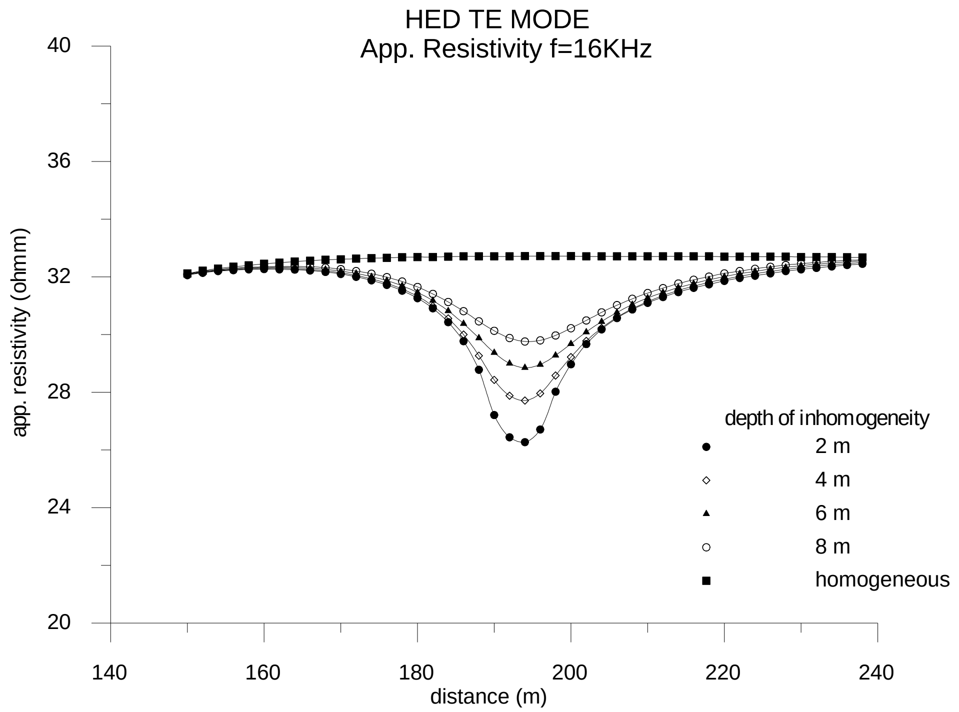
1.Fig. VLF TM mode app. resistivity over conductive inhomogeneity



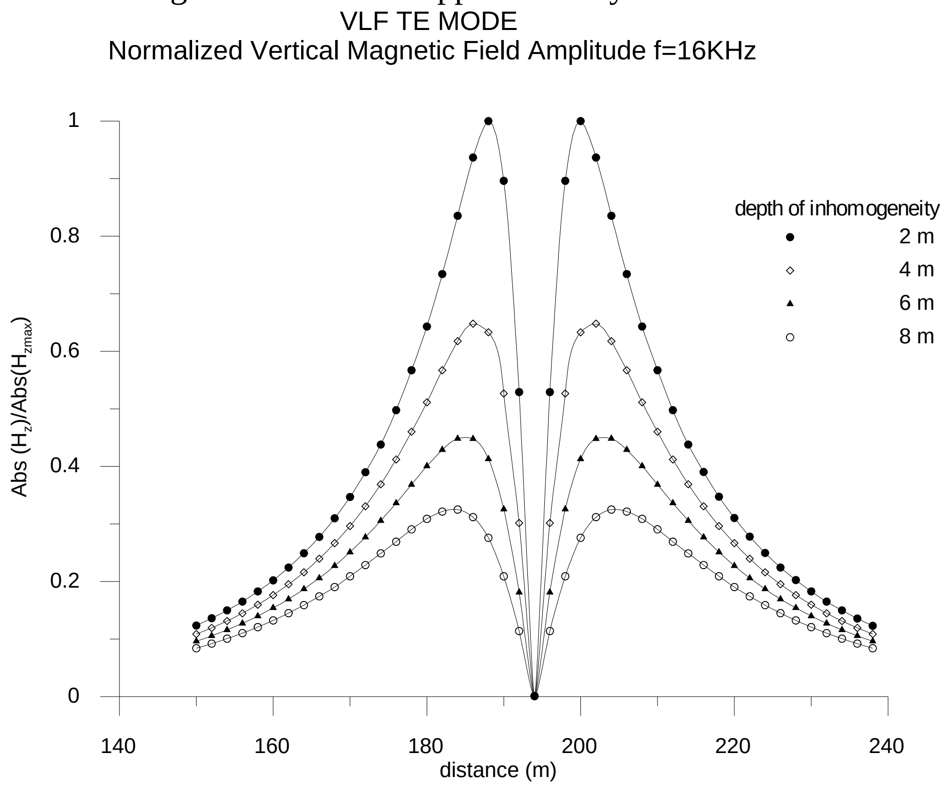
2.Fig. HED TM mode app. resistivity over conductive inhomogeneity



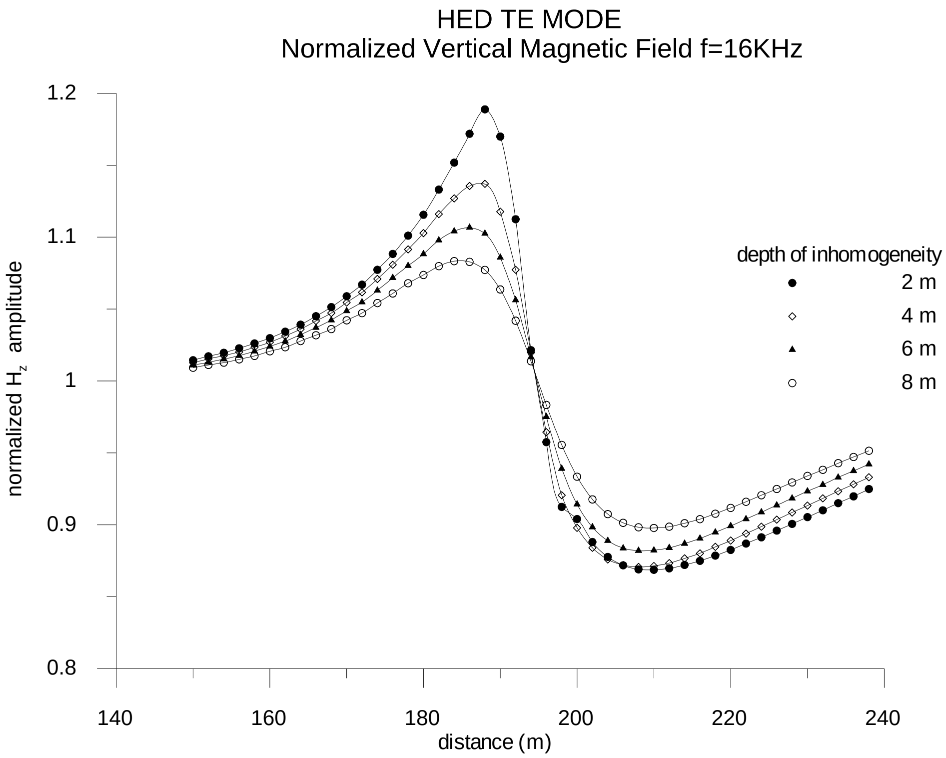
3.Fig. VLF TE mode app. resistivity over conductive inhomogeneity



4.Fig. HED TE mode app. resistivity over conductive inhomogeneity



5.Fig. Normalized vertical magnetic field over conductive inhomogeneity at different depths in VLF TE mode



6.Fig. Normalized vertical magnetic field over conductive inhomogeneity at different depths in HED TE mode

direction) were used to decrease the error coming from the computation of secondary components in mathematical sense. In order to meet this requirement greater grid size has to be applied in the course of the 2.5-D numerical modelling to compare with the 2D procedure. In HED 2.5-D modelling the transmitter receiver distances were 150m ≤ *r*≥ 240m resulting in 7≤ *r/p* ≥11 at f=16KHz, where *p* denotes the skin depth. For VLF and HED excitations in the two polarizations the apparent resistivities were calculated from the absolute value of impedances (given by equations (29)- (32) in [1]) on the basis of Cagniard formula. In case of VLF plane wave excitation symmetrical responses were computed (Fig. 1. and Fig. 3.) opposite to HED excitation (Fig. 2. and Fig. 4.)

The galvanic effect is similarly presented in TM mode for the two excitations. This effect is especially pronounced if the inhomogeneity has a surface outcrop or it is situated very close to the surface. The cause of asymmetry over the symmetrical structure in case of HED source in TM mode in Fig. 2. is the decay of primary field with increasing transmitter receiver distance. In the other polarization in TE mode current channelling is dominant. Its effect on the apparent resistivity is less then that of the galvanic effect of TM mode. The cause of deviation from the homogeneous half-space resistivity value in HED excitations can be the numerical evaluation of secondary components (E_y in TM mode and H_y in TE mode, equations (27) and (25) in [1]). Taking the differences between apparent resistivities of inhomogeneous and homogeneous situations it can be stated that in this *r/p* range both effects are described in HED excitation similarly as in VLF excitation. Additional investigation is suggested for apparent resistivity phase comparison. Assuming this *r/p* range the substitution of

VLF by HED excitation can be accepted and in the shortage of operating transmitter it can be suggested in the point of view of apparent resistivity.
The vertical magnetic field of TE mode in VLF and that of HED excitation were determined according to equations (6) and (26) in [1].The normalized vertical magnetic field amplitudes are presented in Fig. 5. and Fig.6. and the normalization was made to the H_{zmax} amplitude over the inhomogeneity of 2m depth in case of VLF in Fig.5, and to the homogeneous half-space in Fig. 6 assuming HED excitation, respectively. The geological significance of vertical magnetic component is presented by [5]. Just like in magnetotellurics it has structural inhomogeneity origin in VLF, and it is an EM physical quantity preferred in interpretation [6]. The vertical magnetic field component is present over homogeneous half-space excited by HED source on the surface. From the comparison of TE mode numerical modelling results it can be stated that the resolution of vertical magnetic field amplitude is better in case of VLF to compare with that of HED source. Opposite to resistivity measurements the substitution can not be highly recommended, and the resolution of the vertical magnetic field component can be even worse to compare with that of VLF if there is more then one near-surface inhomogeneity.

SUMMARY

In this paper emphasis was put on the comparison of the EM responses over a near surface inhomogeneity due to plane wave and electric dipole excitation. The imbedded inhomogeneity was located at different depths. From the mathematical formalism [1] it is obvious that measurement and modelling data can be compared for the far-field zone. In this paper the EM responses were gained by FD method in both situations. Because the transmitter-receiver distances in case of HED excitation at this frequency of VLF range was large enough the 2.5-D EM modelling was carried out practically in the far field zone. Physically this is the reason that there must have been similarity for the two situations. On the basis of numerical modelling it can be stated that VLF resistivity measurements can be replaced by HED source excitation if the ratio of the transmitter-receiver distance to skin depth covers the range applied here.

ACKNOWLEDGEMENT

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REFERENCES

[1] PETHŐ, G.: **Formalism comparison of 2-D MT and 2.5-D FEM using electric dipole source.** MicroCAD 2005, Section C: Geology, Mineral Resources, Proceedings pp.31-36, Miskolc
[2] ZONGE, K.L., HUGHES,L.J.: **Controlled source audio-frequency magnetotellurics**. In: Electromagnetic methods in applied geophysics, Vol.2. ed. M.N.Nabighian, 1987, SEG
[3] PETHŐ, G., FICSÓR, L.: **Source polarization effect in case of elongated surface inhomogeneities covering transition zone**. 63rd EAGE Conference & Exhibition, Extended Abstracts, 2001, Vol. 2, P050, ISBN 90-73781-18-3, Amsterdam
[4] BÁLINT, T., FICSÓR, L., PETHŐ, G., TÖRÖK, I.: **Some detectability aspects of FEM using HED sources.** 8th European Environmental and Engineering Geophysics Conference, Proceedings, Section New Technologies and Research Trends, 2002, pp. 519-522, Aveiro
[5] TAKÁCS, E.: **The role of the vertical pulsations of the magnetic field in magnetotelluric measurements**. Acta Geodetica, Geophysica et Montanistica Acad. Sci. Hung., 1971, Tom.6 (1-2), pp. 99-110, Budapest
[6] TAKÁCS, E., PETHŐ, G., SZABÓ, I.: **Comparative investigations about the applicability of current density pseudosections in the interpretation of 2D VLF vertical magnetic anomalies.** Acta Geod. Geoph. Hung., Vol.40 (2) pp. 127-146, Budapest

Source polarization effect in case of elongated surface inhomogeneities covering transition zone

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Introduction

Practical CSAMT measurements are frequently made not only in the far-field zone but also in transition regime. For homogeneous earth there is a relatively smooth transition from far-field to near-field behavior. In nonhomogeneous environments this transition zone behavior becomes complex and depends upon the type of configuration, separation and resistivity distribution. Special attention has to be made to resistive basement overlaid by conductive layer model which supports the reduction of plane-wave regime (Wannamaker, 1997). The aim of this paper is to present a numerical modeling of 2.5D EM response and to examine the effects of surface inhomogeneities in TM and TE mode covering the part of the far-field zone neighboring transition zone, the transition zone approaching the near field regime.

Method

To solve 2.5D EM problem numerically the key is to Fourier transform of Maxwell's equations with respect to the structural strike (*x*). For TE mode when electric source parallel to the strike the following equations can be derived:

$$-\frac{\partial}{\partial y}\left(\frac{1}{\zeta^M}\frac{\partial \tilde{H}_x}{\partial y}\right)-\frac{\partial}{\partial z}\left(\frac{1}{\zeta^M}\frac{\partial \tilde{H}_x}{\partial z}\right)-jk_x\frac{\partial \xi}{\partial y}\frac{\partial \tilde{E}_x}{\partial z}+jk_x\frac{\partial \xi}{\partial z}\frac{\partial \tilde{E}_x}{\partial y}+\mathrm{Y}^M\tilde{H}_x=0 \qquad (1)$$

$$-\frac{\partial}{\partial y}\left(\frac{1}{\zeta^E}\frac{\partial \tilde{E}_x}{\partial y}\right)-\frac{\partial}{\partial z}\left(\frac{1}{\zeta^E}\frac{\partial \tilde{E}_x}{\partial z}\right)-jk_x\frac{\partial \xi}{\partial y}\frac{\partial \tilde{H}_x}{\partial z}+jk_x\frac{\partial \xi}{\partial z}\frac{\partial \tilde{H}_x}{\partial y}+\mathrm{Y}^E\tilde{E}_x=-\tilde{i}_{sx} \qquad (2)$$

If the electric source is perpendicular to the strike then:

$$-\frac{\partial}{\partial y}\left(\frac{1}{\zeta^M}\frac{\partial \tilde{H}_x}{\partial y}\right)-\frac{\partial}{\partial z}\left(\frac{1}{\zeta^M}\frac{\partial \tilde{H}_x}{\partial z}\right)-jk_x\frac{\partial \xi}{\partial y}\frac{\partial \tilde{E}_x}{\partial z}+jk_x\frac{\partial \xi}{\partial z}\frac{\partial \tilde{E}_x}{\partial y}+\mathrm{Y}^M\tilde{H}_x=-\frac{\partial}{\partial z}\left(\frac{\tilde{i}_{sy}}{\zeta^M}\right) \qquad (3)$$

$$-\frac{\partial}{\partial y}\left(\frac{1}{\zeta^E}\frac{\partial \tilde{E}_x}{\partial y}\right)-\frac{\partial}{\partial z}\left(\frac{1}{\zeta^E}\frac{\partial \tilde{E}_x}{\partial z}\right)+jk_x\frac{\partial \xi}{\partial y}\frac{\partial \tilde{H}_x}{\partial z}-jk_x\frac{\partial \xi}{\partial z}\frac{\partial \tilde{H}_x}{\partial y}+\mathrm{Y}^E\tilde{E}_x=jk_x\frac{\partial}{\partial y}\left(\xi\tilde{i}_{sy}\right) \qquad (4)$$

where $\xi=\left(k_x^2-k^2\right)^{-1}$, k denotes the wavenumber, and from the transmission sheet formulation $\mathrm{Y}^M=j\omega\mu$ (TM admittance), $\mathrm{Y}^E=(\sigma+j\omega\varepsilon)$ (TE admittance), $\zeta^E=\left(1-k_x^2/k^2\right)\mathrm{Y}^M$ (TE impedance), $\zeta^M=\left(1-k_x^2/k^2\right)\mathrm{Y}^E$ (TM impedance).

The partial differential equations above can be finite differenced with the method of parameter lumping applied by Stoyer (1974). After the solution of the linear systems belonging to different k_x values numerical inverse transform was used to determine the in-strike and off-strike EM field components.

Numerical Model Results

The investigated models and configurations are summarized in Fig. 1. The selected basic model (1D hom.) reflects the geoelectric parameters of a basin area: the resistive (100 ohmm) basement is overlaid by less resistive (20 ohmm) layer with thickness of 1000 m. 1D cond., 1D res. models correspond to the three-layered 1D model with resistivity of top layer 5 ohmm and 80 ohmm, respectively. The inhomogeneous model with a conductive embedding (referred as 2D cond.) in a two-layer earth is shown in Fig. 1. The resistive inhomogeneity (2D res.) has the same geometry as the conductive one. If both the transmitter and receiver electric dipole are parallel to the strike (T_x –R_x array, broadside configuration) TE mode, and if they are orthogonal to it (T_y – R_y array, collinear configuration) TM mode terms are used.

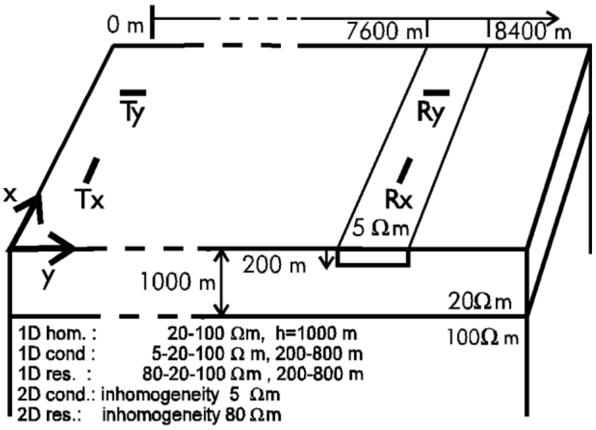

Fig.1. Model description

The EM responses were determined along the transmitter-receiver line perpendicular to the strike with separation interval of 7450 m $\leq R \leq$ 8550 m in the frequency range of 100 Hz $\leq f \leq$ 0,1 Hz. The 2.5-D code was applied to compute 1D EM responses and was compared with the reference responses calculated by linear filter theory (Ádám et al., 1983) for the same 1D hom. model. On the basis of this comparison the error of impedance amplitudes was less than 4.5 per cent and the error of impedance phases was within 2 degrees in this separation and frequency range. To achieve this accuracy 27 wavenumbers with logarithmically equidistant distribution and three grid geometries were needed. Fourier spectrum of EM fields were determined for inhomogeneous cases, and it can be stated that $k_{x\ max}$ = 0.005 m^{-1} was sufficient except at nodes lying at the boundaries. The grid sizes were 129x31, 150x50, 249x50.

Instead of Cagniard resistivities the impedance amplitudes are presented in Figures 2.-5., because only the left part of sounding curves (where the impedances are independent of the configuration for a given separation) reflects the far-field zone. The main characteristics of these impedance sounding curves that they show the presence of resistive basement, because separation distance is greater than the thickness of the overburden, and the EM responses are determined at those frequencies also where the skin depth for the overburden exceeds its thickness. Even in the simplest two-layered basin model (1D hom.) at the receiver the attenuated EM fields propagating in the conductive layer interfere with the waves coming from the resistive basement causing minimum in the impedance soundings. The interference is more complicated if the overburden contains two layers. Characteristics transition notch shoulder develops after the far-field zone if the overburden contains conductive layer. All computed impedance phases show abrupt decrease in the beginning of the transition zone neighboring far-field zone. The presence of top conductive layer or conductive inhomogeneity in collinear configuration results negative phases of impedance. All phases tend to zero with increasing time period approaching near-field regime. The difference between the 2D cond. and 1D hom. impedance responses for broadside configuration (Fig. 2. and 3.) is caused by the excessive current flow along the axis of the inhomogeneity. The induction effect is not so considerable if the inhomogeneity is resistive. This behavior is similar to 2D MT TE mode. For less separation (Fig. 3. and 5.) the frequency range of far-field zone is reduced, resulting in transition notch at higher frequencies. If the overlying layers contain conductive one then the transition notch is more pronounced (1D cond.). In the near-field zone both the electric and magnetic fields are saturated and impedances tends to constants depending upon the separation and the apparent resistivity belonging to DC. For homogeneous half space models with resistive basement they are proportional to the total conductances.

For collinear configuration even in case of homogeneous half-space (5 ohmm $\leq \rho \leq$ 100 ohmm) characteristic transition notch forms within the separation and frequency range investigated here. This notch becomes steeper in the presence of conductive overburden. The two effects can not be separated on the sounding curve, and the upper conductive layer enhances the steepness of the transition zone notch. This stronger tuning effect can be observed in Fig. 4. and 5. For TM node where the skin depth is very large to the size of the inhomogeneity then static effect is experienced. The gradual development of this effect can be observed in the function of frequency. On the conductive side the electric field of the charge distribution reduces the primary electric field making the conductive part even more conductive (Fig. 4., right part of 2D cond. response) and making the resistive part even more resistive (Fig. 5. right part of 2D cond. response). This galvanic effect can be seen in case of resistive inhomogeneity also.

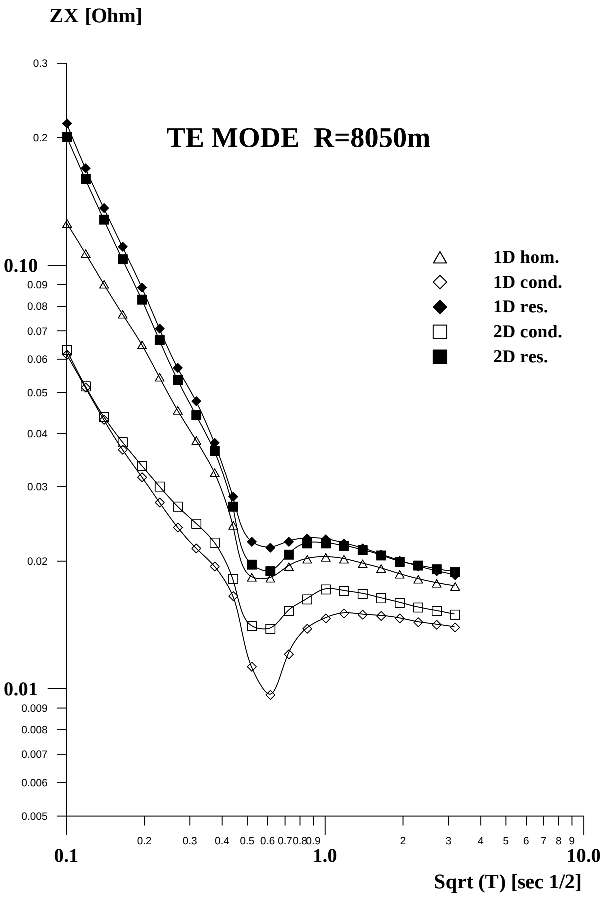

Fig. 2. TE impedances in the inhomogeneity

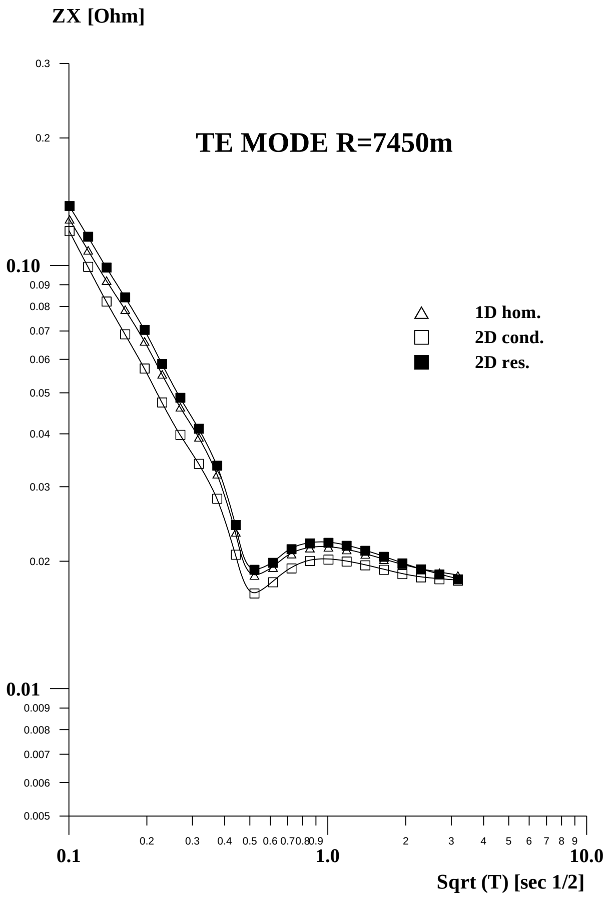

Fig. 3. TE impedances near the boundary

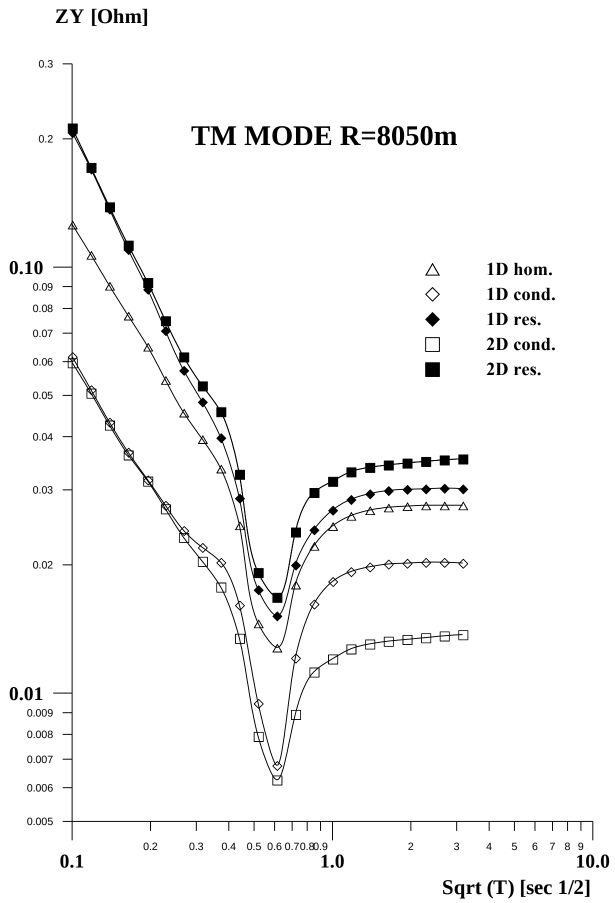

Fig. 4. TM impedances in the inhomogeneity

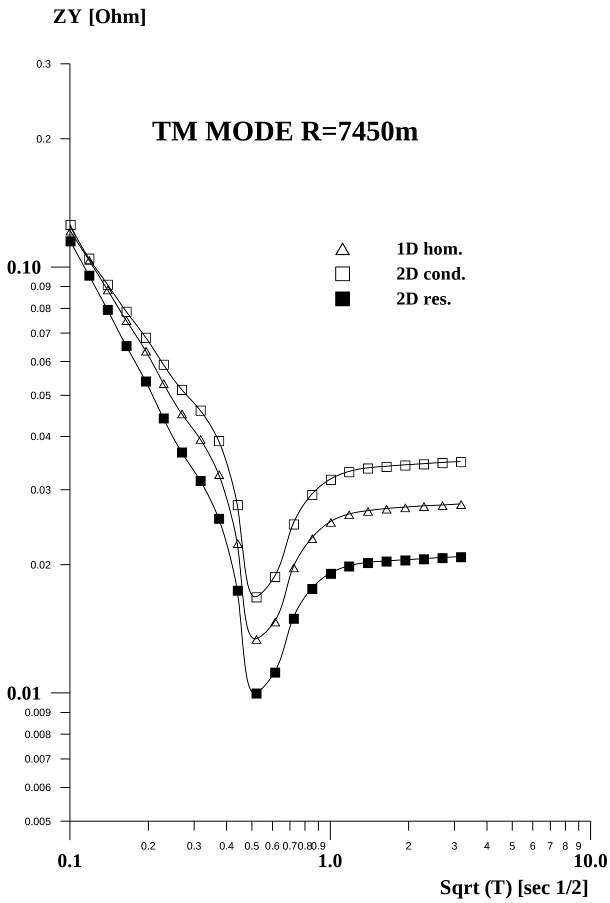

Fig. 5. TM impedances near the boundary

Conclusions

A finite difference method in the (k_x,y,z) domain was applied to present how EM fields are distorted by 2D surface conductivity inhomogeneities in both TM and TE mode. The impedances were determined in the part of far-field zone close to transition zone, in the whole transition zone approaching the near-field regime. For the host 1D model with resistive basement the tuning effect was stronger in the collinear array than in broadside configuration. For the investigated 2D inhomogeneities the impedances of TM and TE mode were reflecting similar tuning effect as 1D models with relatively unchanged position of transition notch. In TE mode EM responses are distorted by current channeling in strike direction. In TM mode significant static shift at lower frequencies covering the range of the transition regime can be experienced. The method presented here can be used to investigate the superposition of these effects in the transition zone and may serve to interpret EM responses over elongated conductivity structure.

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References

[1] Ádám, A., Szarka, L. and Varga, M., 1983, Physical and mathematical modeling of crustal conductivity anomalies in the Pannonian Basin: AGGM Hung., 18. 467-488.

[2] Stoyer, C.H., 1974, Numerical solutions of the response of a 2-D earth to an oscillating magnetic dipole source with application to a groundwater field study, Ph. D. Thesis

[3] Wannamaker, P.E., 1997, Tenzor CSAMT survey over the Sulphur Springs thermal area, Valles Caldera, New Mexico, U.S.A., Part II: Implications for CSAMT methodology, Geophysics, 62, 466-476.

SOME DETECTABILITY ASPECTS OF FEM USING HED SOURCES

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Introduction

Controlled source frequency domain electromagnetic (FEM) methods apply either magnetic or electric dipole sources. In both cases they can work in different zones defined by the product of wavenumber and separation. In this paper some detectability aspects of FEM using horizontal electric dipole source are studied with emphasis on the transition zone and its vicinity. From the interpretation point of view the ideal situation is to measure in the far-field regime for all frequencies at any separation (*r*). For homogeneous earth it fulfils when $r_{\min} > 6\delta$ where δ denotes skin depth. If measurements are also proposed in the transition zone, this restriction has a form of $r_{\min} > 0{,}2\delta$ given by Kaufman and Keller (1983). In these two regimes both electric and magnetic field depend on frequency and resistivity.

Numerical modelling

Even for homogeneous half-space in the transition zone the amplitude of electric field component has a minimum in the function of frequency at a given separation, and the minimum is more pronounced for collinear configuration compared with broadside array. This minimum contributes to the development of transition notch. The relatively smooth transition behaviour of homogeneous earth from far-field to near field zone becomes complex for inhomogeneous environments even for horizontally layered earth. Numerical modelling showed that this transition zone response depends on not only the resistivity distribution but also the type of configuration. For example not far from the source over H-type horizontally stratified half-space if the effect of conductive sandwiched layer and the transition notch appear at the same frequencies, their superposition results in tuning effect on the resistivity sounding curves belonging to collinear configuration (Bálint and Török, 2000). Using the two configurations or other EM methods this masking can be resolved. It was also found by Zonge and Hughes (1991) that the transition notch is more pronounced in areas where a conductive layer lies above a resistive basement. In this situation the source fields enter the basement and may diffuse to the receiver with relatively little attenuation, promoting near-field behaviour as discussed by Wannamaker (1997).

The main goal of this study is to compare TM and TE mode responses in case of elongated underground inhomogeneity hosted by resistive basement when separation-frequency range covers the part of far-field zone neighbouring transition zone, the entire transition zone approaching the near-field regime. In order to do it we apply a 2.5-D finite difference program developed by Pethő and Ficsór (2001) and to enhance the effect of inhomogeneity normalized impedance sections compared with background are presented. This transformation can be useful in detectability. For TE mode (broadside configuration, $T_x - R_x$ array in Fig. 1. when electric source parallel to the structural strike (*x*)) and for TM mode (collinear configuration, $T_y - R_y$ array when horizontal electric dipoles are perpendicular to the strike) two partial differential equation systems were derived and solved in the Fourier domain (Pethő & al., 1995). To avoid unrealistic apparent resistivity changes caused by the application of MT Cagniard resistivity definition for the transition zone and near-field regime the sections are presented in terms of impedance. Other reason of impedance use is that in the three zones *E* and *H* decay as r^{-3} and/or r^{-2}, so the EM fields attenuation with separation can be avoided. On the surface, in the plane perpendicular to the strike and containing the source the impedances and their phases can be given after inverse Fourier transforms:

$$|Z_{TE}| = \frac{|E_x|}{|H_y|} = \frac{\left| \int_0^{k_{x\max}} \tilde{E}_x dk_x \right|}{\left| \int_0^{k_{x\max}} \left(\frac{\frac{\partial \tilde{E}_x}{\partial z}}{\left(k_x^2/k^2 - 1\right) j\omega\mu} - \frac{jk_x}{k_x^2 - k^2} \frac{\partial \tilde{H}_x}{\partial y} \right) dk_x \right|}; \qquad \phi(Z_{TE}) = \arctan(E_x, H_y) \quad (1)$$

$$|Z_{TM}| = \frac{|E_y|}{|H_x|} = \frac{\left| \int_0^{k_{x\max}} \left(\frac{\frac{\partial \tilde{H}_x}{\partial z}}{\left(1 - k_x^2/k^2\right) \omega} - \frac{jk_x}{k_x^2 - k^2} \frac{\partial \tilde{E}_x}{\partial y} \right) dk_x \right|}{\left| \int_0^{k_{x\max}} \tilde{H}_x dk_x \right|}; \qquad \phi(Z_{TM}) = \arctan(E_y, H_x) \quad (2)$$

In these equations k_x stands for the along-strike wavenumber, *k* denotes the wavenumber. The investigated model can be seen in Fig. 1. The applied frequency range was 2.5 MHz >*f* > 1 KHz. The impedance amplitude sections are presented in Fig. 2., and impedance phase sections are in Fig. 3. In both Figures TM mode responses are on the left, and TE mode responses on the right. Normalizations are made here to the two-layered half-space model to recover information from the transition zone which is still sensitive to the frequency and conductivity distribution (to enhance the EM distortion effect caused by the inhomogeneity i.e. to increase detectability, normalizations are suggested to the appropriate 1D model in general case).

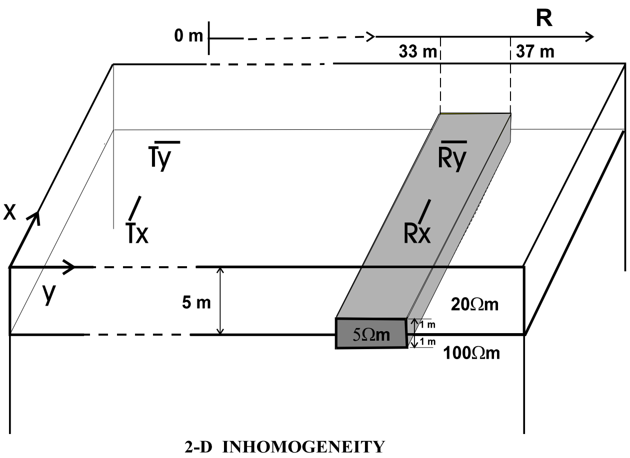

Fig. 1. Model description

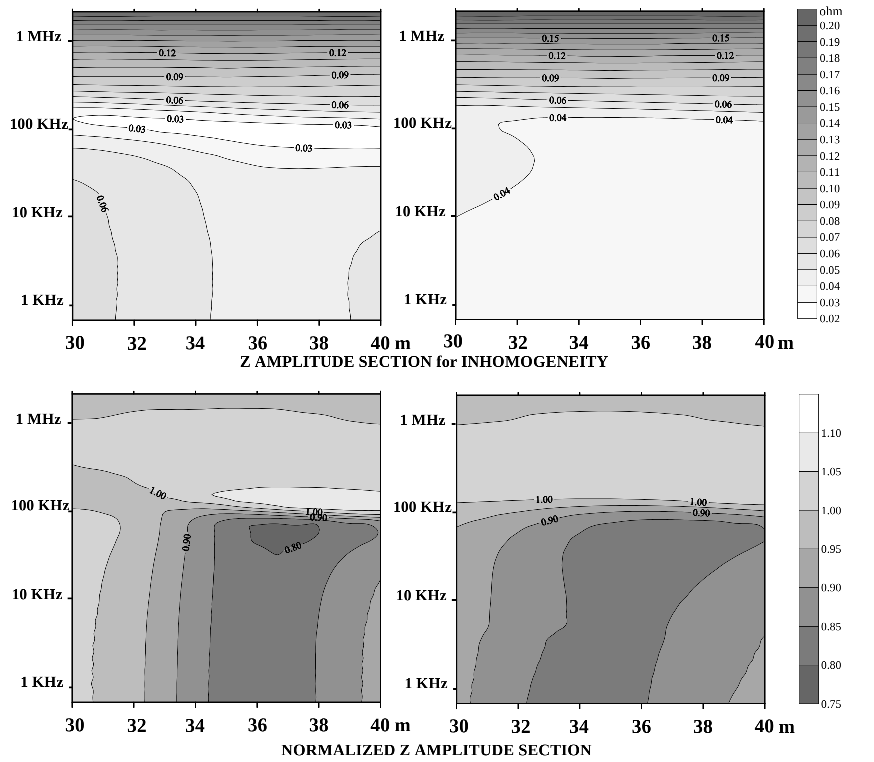

Fig. 2. TM mode (left) and TE mode (right) impedance amplitude and normalized amplitude sections

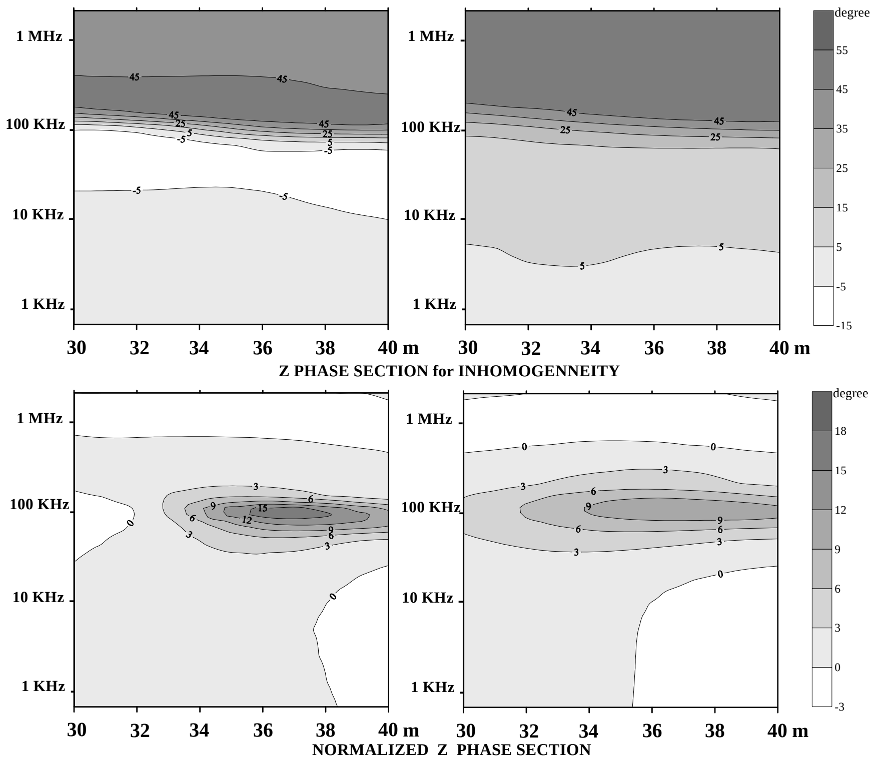

Fig. 3. TM mode (left) and TE mode (right) impedance phase and normalized impedance phase sections

In contrast with 2D MT there is a coupling between the two polarizations. It means that due to the 3D character of the source galvanic effect and current channelling simultaneously can be experienced in both cases for the hosted conductive elongated inhomogeneity. In this situation – similarly to 2D MT TM mode – the TM mode has a better resolution than TE mode in term of impedance amplitude due to the pronounced galvanic effect. Even for homogeneous two-layer case extreme slopes of impedances can be observed for collinear configuration and transition notch developes. Because impedance phase is proportional to the derivative of resistivity, negative phases can be observed in the collinear array. For the purpose of imaging impedance phase measurements are even more suggested. However, accurate measurements (very high signal to noise ratio, dense sampling in frequency domain) are needed because the greatest EM field changes can be observed in the transition zone. It can be also stated that this transformation in the separation and frequency domain can not be proposed if the inhomogeneity situates between the transmitter and receiver (i.e. the inhomogeneity is not under the receivers). Normalized impedance (including amplitude and phase) seems to be a reliable parameter to recover information of transition zone and enhance the EM distortion effect of the inhomogeneity if the series of receivers completely covers the near-surface inhomogeneity.

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References

Bálint, T., and Török, I., Some numerical modelling results of geophysical methods aiming at environmental protection. International Computer Science Conference, Miskolc, Section Geoinformatics, Environment Protection, Proceedings, pp. 25-31, 2000.

Kaufman, A., A., and Keller, G., V., Frequency and transient soundings. Elsevier Science Publishers, 1983.

Pethő, G., Kaikkonen, P., and Vanyan, L.L., Numerical modelling for the effect of a 2-D seafloor trench on sea-bottom EM measurements using horizontal electric dipole sources, Geophysica, 31, 1-21, 1995.

Pethő, G. and Ficsór, L. Source polarization effect in case of elongated surface inhomogeneities covering transition zone. 63rd EAGE Amsterdam, Extended Abstracts, Vol. 2, P050, 2001.

Wannamaker, P., E., Tenzor CSAMT survey over the Sulphur Springs thermal area, Valles Caldera, New Mexio, U.S.A., Part II: Implications for CSAMT methodology, Geophysics, 62, 466-476, 1997.

Zonge, K., L., and Hughes, Controlled source audiomagnetotellurics, in M.N. Nabighian, Ed., Electromagnetic methods in applied geophysics, 2B, Application, Soc. Expl. Geophys., 713-809, 1991.

FEM Source Effect Investigation with 2.5D Numerical Modelling

If there is a horizontal electrical dipole (HED) source the closeness of the source results in non-planewave effect at the receiver site. This source effect cannot be observed in the apparent resistivity of the far-field zone because both EM components have $1/r^3$ dependence. In the transition zone *E* is proportional to $1/r^3$, while *H* offset dependence gradually changes into $1/r^2$ approaching the near-field zone. In the near-field zone *E* is proportional to $1/r^3$ and *H* decays as $1/r^2$ and neither EM component has frequency dependence. This non-planewave effect manifests itself in different ways at collinear and at broadside configurations. For example in the transition zone the amplitude of electric field component has a minimum in the function of frequency at a given separation, and the minimum is more pronounced for the collinear configuration compared with the broadside array. This minimum contributes to the development of transition notch. The source polarization effect was presented by Pethő and Ficsór (2001) for elongated surface inhomogeneities under the receiver site. The development of shadow and source overprint effects is due to the conductivity inhomogeneity between the source and receiver site (Zonge and Hughes, 1991). In this paper the source effect is referred to as source overprint effect if there is a conductivity inhomogeneity directly under the source, and it is referred to as shadow effect when the inhomogeneity is not situated under the source, but it is between the transmitter and receiver site. In the paper these effects are investigated for elongated conductivity structures with HED sources treated as point sources.

2.5D numerical modelling of source effects

Opposite the MT fields in the 2.5D situation the two modes cannot be separated. Here TE mode denotes the situation when the direction of electric dipole source is parallel to the strike, and TM mode corresponds to the case when it is perpendicular to it. If $\vec{j}_s = Id\vec{s}\,\delta(\vec{r})$, a point source approximation can be applied. Following the procedure of Stoyer and Greenfield (1976) two partial differential equation systems can be derived for the two polarizations in the strike directional wave number (k_x) domain, which are (1), (2) for the TE mode and (3), (4) for the TM mode:

$$-\frac{\partial}{\partial y}\left(\frac{1}{\zeta^M}\frac{\partial \tilde{H}_x}{\partial y}\right)-\frac{\partial}{\partial z}\left(\frac{1}{\zeta^M}\frac{\partial \tilde{H}_x}{\partial z}\right)-ik_x\frac{\partial \xi}{\partial y}\frac{\partial \tilde{E}_x}{\partial z}+ik_x\frac{\partial \xi}{\partial z}\frac{\partial \tilde{E}_x}{\partial y}+\upsilon^M\tilde{H}_x=0 \qquad (1)$$

$$-\frac{\partial}{\partial y}\left(\frac{1}{\zeta^E}\frac{\partial \tilde{E}_x}{\partial y}\right)-\frac{\partial}{\partial z}\left(\frac{1}{\zeta^E}\frac{\partial \tilde{E}_x}{\partial z}\right)+ik_x\frac{\partial \xi}{\partial y}\frac{\partial \tilde{H}_x}{\partial z}-ik_x\frac{\partial \xi}{\partial z}\frac{\partial \tilde{H}_x}{\partial y}+\upsilon^E\tilde{E}_x=-\tilde{j}_{sx} \qquad (2)$$

$$-\frac{\partial}{\partial y}\left(\frac{1}{\zeta^M}\frac{\partial \tilde{H}_x}{\partial y}\right)-\frac{\partial}{\partial z}\left(\frac{1}{\zeta^M}\frac{\partial \tilde{H}_x}{\partial z}\right)-ik_x\frac{\partial \xi}{\partial y}\frac{\partial \tilde{E}_x}{\partial z}+ik_x\frac{\partial \xi}{\partial z}\frac{\partial \tilde{E}_x}{\partial y}+\upsilon^M\tilde{H}_x=-\frac{\partial}{\partial z}\left(\frac{\tilde{j}_{sy}}{\zeta^M}\right) \qquad (3)$$

$$-\frac{\partial}{\partial y}\left(\frac{1}{\zeta^E}\frac{\partial \tilde{E}_x}{\partial y}\right)-\frac{\partial}{\partial z}\left(\frac{1}{\zeta^E}\frac{\partial \tilde{E}_x}{\partial z}\right)+ik_x\frac{\partial \xi}{\partial y}\frac{\partial \tilde{H}_x}{\partial z}-ik_x\frac{\partial \xi}{\partial z}\frac{\partial \tilde{H}_x}{\partial y}+\upsilon^E\tilde{E}_x=ik_x\frac{\partial}{\partial y}\left(\xi\,\tilde{j}_{sy}\right) \qquad (4)$$

where $k^2=-i\varpi\mu\sigma$, $\zeta^M=\sigma\,(1-\frac{k_x^2}{k^2})$, $\zeta^E=i\omega\mu\,(1-\frac{k_x^2}{k^2})$, $\upsilon^M=i\omega\mu$, $\upsilon^E=\sigma$. In course of FD modelling the electric source term perpendicular to the strike was treated as a distributed parameter in a rectangular grid element. The investigated models are summarized in **Fig.1.**

The applied frequency range was $268.3Hz \geq f \geq 0.1Hz$. In the course of FD modelling three grids (50x249, 45x180, 42x128) and 27 strike directional wavenumbers were applied.

Numerical inverse Fourier transforms were applied to determine the spatial solutions. In this way we received an agreement within 2.5% for the impedance amplitude sounding curves, and an agreement within 1.5^0 for impedance phase frequency sounding curves calculated by 1D and 2.5D codes over the two-layer half-space model of Fig.1. Three 2D models will be investigated in the paper. If there is only one hosted conductive block, it is either directly under the source or it is under the receiver at greater depth. The most complicated model involves both blocks of 5ohmm. To test the 2.5D code reciprocity theorem was also applied. The EM responses were determined with interchanged positions of transmitter and receiver as well over the model with embedded conductivity block under the source. For the two "measurements" the difference between the electric field component amplitude sounding curves was not more than 0.7%, and the difference between the electric field component phase sounding curve was less than 0.2^0 for the two modes.

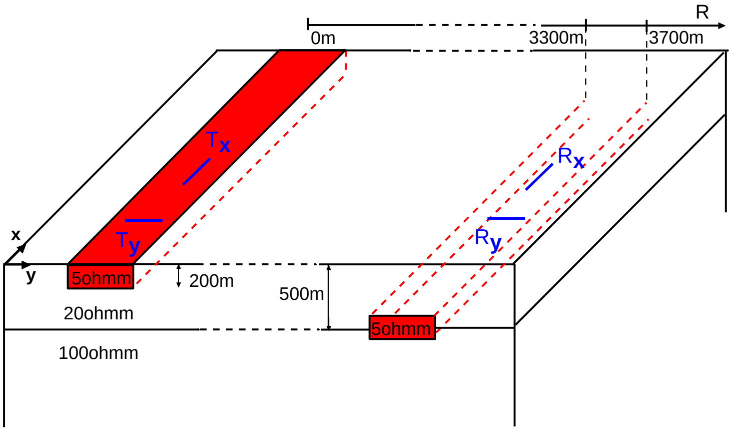

Fig.1. Two-layer half-space with conductive embeddings for source effects investigation

To enhance the effects of inhomogeneities -including source effects- normalized impedance amplitude and normalized impedance phase sections are presented for TE (T_x-R_x array) and TM (T_y-R_y array) mode. The normalization (ratio between impedance amplitudes, and difference between impedance phases) was made to the EM responses of the two-layer half-space. If there is only one inhomogeneity under the source, a gradually decreasing source overprint effect with increasing R can be recognized in **Fig.2.** This effect is approximatly 5 times greater for TM mode than for TE mode in this case.

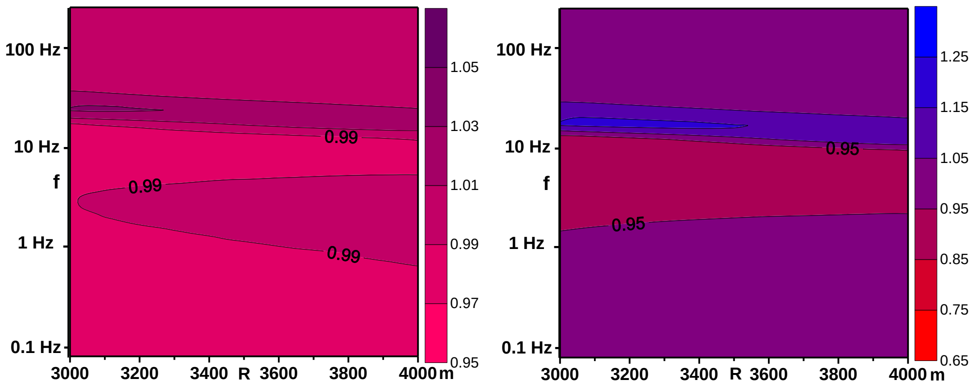

Fig.2. Normalized impedance amplitude sections for TE mode (left) and TM mode (right) over the model with conductive inhomogeneity under the transmitter

The shadow effect for the hosted conductive body under the receiver develops because of the stronger attenuation of the diffusive EM fields in the conductive inhomogeneity. On the basis of the normalized impedance amplitude and impedance phase sections horizontal delineation cannot yield good result for the side of the elongated block away from the source (at 3700m). The shadow effect can be observed both in the normalized impedance amplitude and phase sections and in both modes in **Fig.3.** and in **Fig.4.** In the TE mode mainly the current channelling and in the TM mode the galvanic effect is dominant. Here the shadow effect distorts the frequency sounding curves from the right side of the inhomogeneity with increasing transmitter-receiver range. The use of transmitter on the opposite side of the inhomogeneity may help in the EM interpretation.

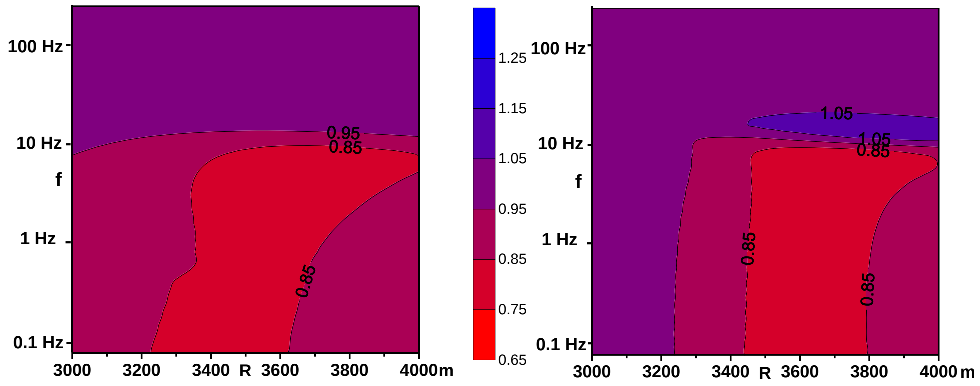

Fig.3. Normalized impedance amplitude sections for TE mode (left) and TM mode (right) over the model with conductive inhomogeneity under the receiver

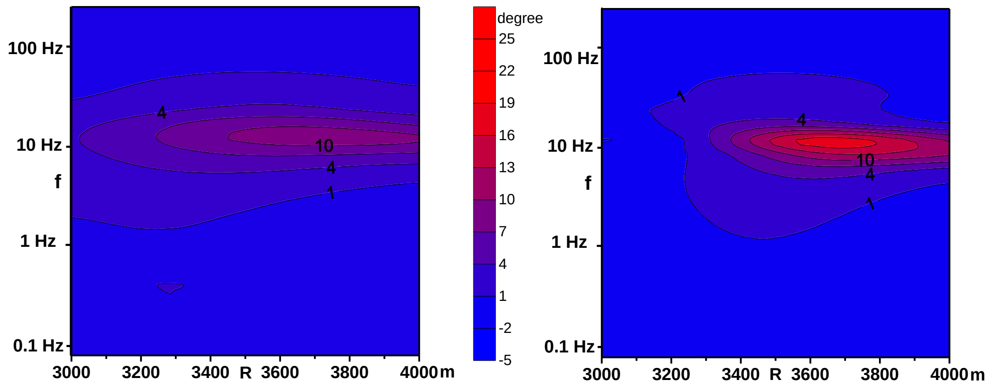

Fig.4. Normalized impedance phase sections for TE mode (left) and TM mode (right) over the model with conductive inhomogeneity under the receiver

The EM responses in TE mode are smoother than in TM mode due to the continuity of strike directional electric field component. The impedance phase (i.e. the phase of apparent resistivity) section delineates better the inhomogeneity than the impedance amplitude section especially in TM mode. For relatively simple cases (there is only one inhomogeneity or they are distant from each other) this kind of "imaging" may be useful and it is suggested for both polarizations.

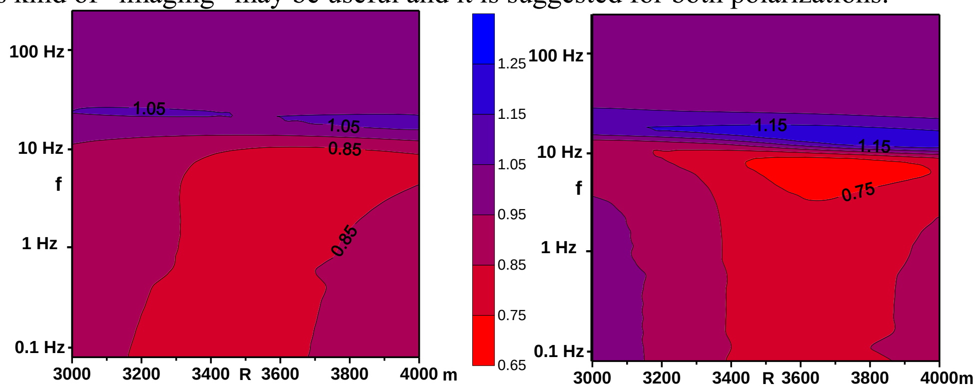

Fig.5. Normalized impedance amplitude sections for TE mode (left) and TM mode (right) over the model with conductive inhomogeneities under the source and the receiver

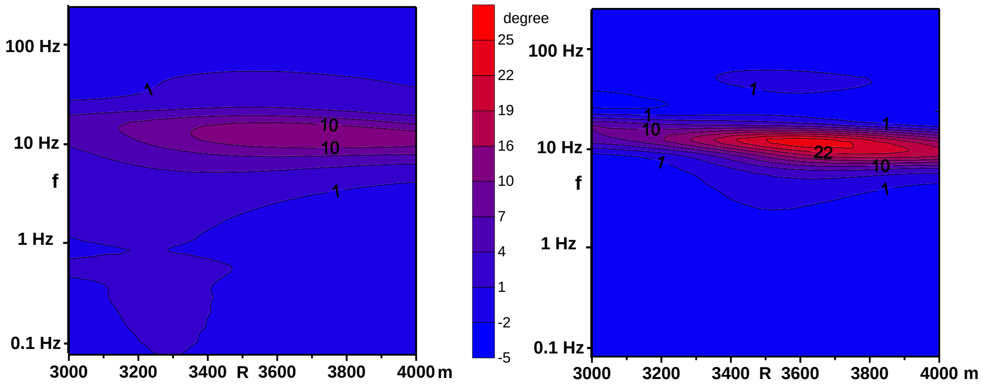

Fig.6. Normalized impedance phase sections for TE mode (left) and TM mode (right) over the model with conductive inhomogeneities under the source and the receiver

The investigated models with a conductive inhomogeneity (either under the source or the receiver) can be characterized by mainly impedance amplitude decrease and positive impedance phase shift to compare with the reference model. In the presence of the two inhomogeneities the superposition of the EM responses can be observed in **Fig.5.** and in **Fig.6.** Because both embedded inhomogeneities are conductive, additional impedance amplitude decrease and impedance phase increase can be experienced to compare with the homogeneous case. It is typical that source overprint effect (due to the inhomogeneity under the source) flattens out the EM responses of the embedding inhomogeneity (between the source and receiver) horizontally.

Conclusions

The developed 2.5D finite difference forward modelling code makes it possible to investigate the different source effects in broadside and in collinear configurations as well. The investigations covered the transition zone. To present these effects impedance amplitude and impedance phase sections were applied instead of apparent resistivity sections. The normalized impedance phase sections delineate the inhomogeneity better than the impedance amplitude sections. For the investigated models the shadow effect develops similarly in the two modes and greater source overprint and non-planewave effect were observed in TM mode than in TE mode. These source effects manifest themselves characteristically in the case of simple 2D structures, so numerical modelling may be a useful tool in the interpretation of the multi-receiver FEM measurements using HED sources. The statements of this paper can be applied for the models with the same *kL* (*L* represents a linear dimension in the model) dimensionless parameter till displacement currents can be neglected.

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References

Pethő, G., Ficsór, L., Source polarization effect in case of elongated surface inhomogeneities covering transition zone. 63rd EAGE Amsterdam, Extended Abstracts, Vol. 2, P050, 2001.
Stoyer, C., H., Greenfield, R., J., Numerical solutions of the response of a two-dimensional earth to an oscillating magnetic dipole source, Geophysics, 41., 519-530, 1976.
Zonge, K., L., and Hughes, L.,J., Controlled source audiomagnetotellurics, in M.N. Nabighian, Ed., Electromagnetic methods in applied geophysics, 2B, Application, Soc. Expl. Geophys., 713-809, 1991.

DIFFERENTIAL EQUATIONS OF FEM USING ELECTRIC DIPOLE SOURCE FOR ELONGATED STRUCTURES WITH CONDUCTIVITY ANISOTROPY

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Abstract. In this paper the most important relationships are derived for frequency domain 2.5D situation when the electric source is either parallel or perpendicular to the strike of the anisotropic conductivity structure. From the Fourier transforms of the Maxwell's equations coupled partial differential equations are derived, in which besides the wave number, frequency and the EM rock properties only the strike direction EM field components and their partial derivatives in the wave number (k_x) domain can be found. On the basis of comparison between differential equations for isotropic and anisotropic case it can be stated that opposite the isotropic situation second order mixed partial derivative of strike directional Fourier EM field component can be found in the anisotropic case and except the coefficient of the strike directional Fourier magnetic field component all distributed parameters reflect the effect of anisotropy. For the solution of the resulting FD linear equation system instead of the blocktridiagonal LU decomposition the LU decomposition of band matrix is suggested because in contrast with isotropic case 9 point FD scheme is needed. Finally the equations for the spatial solutions of EM field components are presented.

Keywords: frequency domain electromagnetics, elongated structures, anisotropy, wave number domain, FD method, inverse Fourier transformation.

1. Introduction

The geophysical electromagnetic methods apply either natural or artificial source(s) [1], [2]. If the investigated conductivity structure is elongated and the source field is an electric dipole source, the problem to be solved is a 3D one. If the EM source which can be treated mathematically as a point source, we encounter a 2D model exited by a 3D source and this situation is called a 2.5D problem. In the course of EM investigation on elongated conductivity structures exited by electric dipole source special emphasis has to be put on two modes. Distinction is made between TE mode, when the electric source field is parallel to the structural strike, and TM mode, when the electric source field is perpendicular to it. Assuming elongated isotropic conductivity inhomogeneities exited by electric dipole source the partial differential equations were presented by Pethő [3] and besides the investigation of some effects, different numerical model studies were also carried out [4], [5], [6], [7], [8], [9]. In this study the 2.5D FEM partial differential equations are derived for structures with conductivity anisotropy.

2. 2.5D problem solution in wave number domain

In the frequency domain electromagnetic methods $e^{j\omega t}$ time dependent source term is assumed and let x coincide with the strike direction. The basic relationships governing the electromagnetic phenomenon are the Maxwell's equations. One Maxwell's equation expresses that electric fields result from time-varying magnetic induction:

$$rot\vec{E} = -\partial\vec{B}/\partial t \qquad \mathbf{1}$$

$\vec{i}_s = Id\vec{s}\,\delta(\vec{r}\,)$ makes possible a point source approximation. Involving it into the other Maxwell's equation:

$$rot\vec{H} = \vec{i} + \vec{i}_s \qquad \mathbf{2}$$

It represents the fact that the magnetic fields are caused by conduction and source currents. Frequency domain EM methods usually apply EM fields at low frequencies and the conduction currents dominate over displacement currents. For this reason the effect of displacement currents in the second equation was not taken into account.

Generally, the conductivity determining the relationship between the current density vector and the electric field vector must be a tensor (it is a scalar quantity only for isotropic case). The conductivity tensor has a simple form if two of the orthogonal coordinate directions are selected to lie in the direction of maximum conductivity and the third one into the minimum conductivity. Assuming horizontal layering and the same σ_t conductivity in the x and y direction and σ_n conductivity value in the z direction we can write:

$$\sigma = \begin{vmatrix} \sigma_{xx} & 0 & 0 \\ 0 & \sigma_{yy} & 0 \\ 0 & 0 & \sigma_{zz} \end{vmatrix} = \begin{vmatrix} \sigma_t & 0 & 0 \\ 0 & \sigma_t & 0 \\ 0 & 0 & \sigma_n \end{vmatrix} \qquad 3$$

In the case of magnetic source Stoyer and Greenfield recommended that the solution of the original 3D problem could be replaced by the series of solutions of 2D problem in the k_x wave number domain [10]. To follow this procedure the Maxwell's equations (1) and (2) are Fourier-transformed. The Fourier- transformation can be expressed as:

$$\hat{F}(k_x, y, z) = \int_{-\infty}^{\infty} F(x, y, z) e^{-jk_x x} dx \qquad 4$$

In the course of the transformation the Fourier transform of the partial derivative of the EM field components with respect x is also needed:

$$\int_{-\infty}^{\infty} \frac{\partial F(x,y,z)}{\partial x} e^{-jk_x x} dx = \left[F(x,y,z) e^{-jk_x x} \right]_{-\infty}^{+\infty} + jk_x \int_{-\infty}^{\infty} F(x,y,z) e^{-jk_x x} dx = jk_x \hat{F}(k_x, y, z) \qquad 5$$

If x tends to either $+\infty$ or - ∞ then any field component approximates zero due to the finite energy of the source, for this reason the value of expression in brackets must be zero. It follows from (5) that the partial derivative of any EM field components with respect x is proportional the Fourier transform of the component itself. Taking into account this statement and the harmonic time dependence of EM fields, the Fourier transforms of (1) Maxwell's equation in term of component equations we can write as:

$$\frac{\partial \hat{E}_z}{\partial y} - \frac{\partial \hat{E}_y}{\partial z} = -j\omega\mu \hat{H}_x \qquad 6$$

$$\frac{\partial \hat{E}_x}{\partial z} - jk_x \hat{E}_z = -j\omega\mu \hat{H}_y \qquad 7$$

$$jk_x \hat{E}_y - \frac{\partial \hat{E}_x}{\partial y} = -j\omega\mu \hat{H}_z \qquad 8$$

Similarly, if we assume that only horizontal electric dipole source (either in x or y direction) is applied, the Fourier transforms of (2) can be expressed as:

$$\frac{\partial \hat{H}_z}{\partial y} - \frac{\partial \hat{H}_y}{\partial z} = \sigma_t \hat{E}_x + \hat{i}_{sx} \qquad 9$$

$$\frac{\partial \hat{H}_x}{\partial z} - jk_x \hat{H}_z = \sigma_t \hat{E}_y + \hat{i}_{sy} \qquad 10$$

$$jk_x \hat{H}_y - \frac{\partial \hat{H}_x}{\partial y} = \sigma_n \hat{E}_z \qquad 11$$

The aim of the following derivation is to provide partial differential equation systems with equations containing only strike direction EM field components in the wave number (k_x) domain. Separate partial differential equation system will describe the case of electric dipole source of structural strike direction, which is called TE mode, and

another partial differential equation system is valid for the source term perpendicular to the structural strike which is referred as TM mode.
From equations (7) and (11) $j\hat{H}_y$ can be determined and taking them to be equal we get:

$$\sigma_n(1-k_x^2/k_n^2)\hat{E}_z = -\frac{\partial \hat{H}_x}{\partial y} - \frac{k_x}{\omega\mu}\frac{\partial \hat{E}_x}{\partial z} \qquad \textbf{12}$$

Introducing

$$q_n = 1/(1-k_x^2/k_n^2) \qquad \textbf{13}$$

and using the resistivity instead of conductivity from (12)

$$\hat{E}_z = -\rho_n q_n \frac{\partial \hat{H}_x}{\partial y} - \frac{k_x \rho_n q_n}{\omega\mu}\frac{\partial \hat{E}_x}{\partial z} \qquad \textbf{14}$$

From equations (8) and (10) $j\hat{H}_z$ can be determined and taking them to be equal we get:

$$-\sigma_t(1-k_x^2/k_t^2)\hat{E}_y = -\frac{\partial \hat{H}_x}{\partial z} + \frac{k_x}{\omega\mu}\frac{\partial \hat{E}_x}{\partial y} + \hat{i}_{sy} \qquad \textbf{15}$$

Introducing

$$q_t = 1/(1-k_x^2/k_t^2) \qquad \textbf{16}$$

and using the resistivity instead of conductivity from (15)

$$\hat{E}_y = \rho_t q_t \frac{\partial \hat{H}_x}{\partial z} - \frac{k_x \rho_t q_t}{\omega\mu}\frac{\partial \hat{E}_x}{\partial y} - \rho_t q_t \hat{i}_{sy} \qquad \textbf{17}$$

The non-strike directional magnetic field components in the wave number domain can be determined similarly. From equations (8) and (10) $\hat{E}_y$ can be expressed and taking them to be equal we get:

$$-j\omega\mu(1-k_x^2/k_t^2)\hat{H}_z = -\frac{\partial \hat{E}_x}{\partial y} + \frac{jk_x}{\sigma_t}\frac{\partial \hat{H}_x}{\partial z} - \frac{jk_x}{\sigma_t}\hat{i}_{sy} \qquad \textbf{18}$$

From (18) $\hat{H}_z$ can be determined:

$$\hat{H}_z = -\frac{jq_t}{\omega\mu}\frac{\partial \hat{E}_x}{\partial y} - \frac{k_x \rho_t q_t}{\omega\mu}\frac{\partial \hat{H}_x}{\partial z} + \frac{k_x \rho_t q_t}{\omega\mu}\hat{i}_{sy} \qquad \textbf{19}$$

From equations (7) and (11) $\hat{E}_z$ can be expressed and taking them to be equal we receive:

$$j\omega\mu(1-k_x^2/k_n^2)\hat{H}_y = -\frac{\partial \hat{E}_x}{\partial z} - \frac{jk_x}{\sigma_n}\frac{\partial \hat{H}_x}{\partial y} \qquad \textbf{20}$$

From this equation $\hat{H}_y$ has the following form:

$$\hat{H}_y = \frac{jq_n}{\omega\mu}\frac{\partial \hat{E}_x}{\partial z} - \frac{k_x \rho_n q_n}{\omega\mu}\frac{\partial \hat{H}_x}{\partial y} \qquad \textbf{21}$$

3. Partial differential equation systems for TE and TM mode

TE mode is the situation when only an electric dipole source of strike direction is applied. Substituting (19) and (21) into (9) with the assumption of $\hat{i}_{sy} = 0$ in (19) we get (22). The other equation for TE mode (23) can be derived by the substitution of (14) and (17) into equation (6) with the assumption of $\hat{i}_{sy} = 0$ in (17):

$$\frac{j}{\omega\mu}\frac{\partial}{\partial y}\left(q_t\frac{\partial \hat{E}_x}{\partial y}\right)+\frac{k_x}{\omega\mu}\frac{\partial}{\partial y}\left(\rho_t q_t\frac{\partial \hat{H}_x}{\partial z}\right)+\frac{j}{\omega\mu}\frac{\partial}{\partial z}\left(q_n\frac{\partial \hat{E}_x}{\partial z}\right)-\frac{k_x}{\omega\mu}\frac{\partial}{\partial z}\left(\rho_n q_n\frac{\partial \hat{H}_x}{\partial y}\right)+\sigma_t\hat{E}_x=-\hat{i}_{sx} \qquad \textbf{22}$$

$$\frac{\partial}{\partial y}\left(\rho_n q_n\frac{\partial \hat{H}_x}{\partial y}\right)+\frac{k_x}{\omega\mu}\frac{\partial}{\partial y}\left(\rho_n q_n\frac{\partial \hat{E}_x}{\partial z}\right)+\frac{\partial}{\partial z}\left(\rho_t q_t\frac{\partial \hat{H}_x}{\partial z}\right)-\frac{k_x}{\omega\mu}\frac{\partial}{\partial z}\left(\rho_t q_t\frac{\partial \hat{E}_x}{\partial y}\right)-j\omega\mu\hat{H}_x=0 \qquad \textbf{23}$$

In ***TM mode*** the electric dipole source is perpendicular to the structural strike. In order to get the TM mode equations the same steps are needed as in TE mode, however, here $\hat{i}_{sx} = 0$ in (9) and $\hat{i}_{sy} \neq 0$ in (19). Substituting (19) and (21) into (9) we receive (24). The other TM mode equation is (25) which can be derived by the substitution of (14) and (17) into equation (6):

$$\frac{j}{\omega\mu}\frac{\partial}{\partial y}\left(q_t\frac{\partial \hat{E}_x}{\partial y}\right)+\frac{k_x}{\omega\mu}\frac{\partial}{\partial y}\left(\rho_t q_t\frac{\partial \hat{H}_x}{\partial z}\right)+\frac{j}{\omega\mu}\frac{\partial}{\partial z}\left(q_n\frac{\partial \hat{E}_x}{\partial z}\right)-\frac{k_x}{\omega\mu}\frac{\partial}{\partial z}\left(\rho_n q_n\frac{\partial \hat{H}_x}{\partial y}\right)+\sigma_t\hat{E}_x=\frac{k_x}{\omega\mu}\frac{\partial}{\partial y}\left(\rho_t q_t\hat{i}_{sy}\right) \qquad \textbf{24}$$

$$\frac{\partial}{\partial y}\left(\rho_n q_n\frac{\partial \hat{H}_x}{\partial y}\right)+\frac{k_x}{\omega\mu}\frac{\partial}{\partial y}\left(\rho_n q_n\frac{\partial \hat{E}_x}{\partial z}\right)+\frac{\partial}{\partial z}\left(\rho_t q_t\frac{\partial \hat{H}_x}{\partial z}\right)-\frac{k_x}{\omega\mu}\frac{\partial}{\partial z}\left(\rho_t q_t\frac{\partial \hat{E}_x}{\partial y}\right)-j\omega\mu\hat{H}_x=\frac{\partial}{\partial z}\left(\rho_t q_t\hat{i}_{sy}\right) \qquad \textbf{25}$$

4.Comparison between the isotropic and anisotropic case

If we make a comparison between (22)-(23) and (24)-(25) it is obvious that there are differences in the inhomogeneous terms (i.e. differences between the right hand side of equations (22) and (24); (23) and (25), respectively). The derivatives of the inhomogeneous terms have to be taken in the case of TM mode opposite the TE mode. The situation is similar to the isotropic case, the only difference is that the longitudinal apparent resistivity has to be taken into account in the vicinity of the source perpendicular to the strike. Additional difference between the partial differential equations of the isotropic and anisotropic case can be found in the left hand sides of equations for both TE mode and TM mode. The TE mode equations for isotropic situation are as follows [3]:

$$-\frac{\partial}{\partial y}\left(\frac{1}{\zeta^E}\frac{\partial \hat{E}_x}{\partial y}\right)-\frac{\partial}{\partial z}\left(\frac{1}{\zeta^E}\frac{\partial \hat{E}_x}{\partial z}\right)+jk_x\frac{\partial\xi}{\partial y}\frac{\partial \hat{H}_x}{\partial z}-jk_x\frac{\partial\xi}{\partial z}\frac{\partial \hat{H}_x}{\partial y}+\upsilon^E\hat{E}_x=-\hat{i}_{sx} \qquad \textbf{26}$$

$$-\frac{\partial}{\partial y}\left(\frac{1}{\zeta^M}\frac{\partial \hat{H}_x}{\partial y}\right)-\frac{\partial}{\partial z}\left(\frac{1}{\zeta^M}\frac{\partial \hat{H}_x}{\partial z}\right)-jk_x\frac{\partial\xi}{\partial y}\frac{\partial \hat{E}_x}{\partial z}+jk_x\frac{\partial\xi}{\partial z}\frac{\partial \hat{E}_x}{\partial y}+\upsilon^M\hat{H}_x=0 \qquad \textbf{27}$$

The TM mode equations for isotropic case are different from (26) and (27) only in the source terms:

$$-\frac{\partial}{\partial y}\left(\frac{1}{\zeta^E}\frac{\partial \hat{E}_x}{\partial y}\right)-\frac{\partial}{\partial z}\left(\frac{1}{\zeta^E}\frac{\partial \hat{E}_x}{\partial z}\right)+jk_x\frac{\partial\xi}{\partial y}\frac{\partial \hat{H}_x}{\partial z}-jk_x\frac{\partial\xi}{\partial z}\frac{\partial \hat{H}_x}{\partial y}+\upsilon^E\hat{E}_x=jk_x\frac{\partial}{\partial y}\left(\xi\,\hat{i}_{sy}\right) \qquad \textbf{28}$$

$$-\frac{\partial}{\partial y}\left(\frac{1}{\zeta^M}\frac{\partial \hat{H}_x}{\partial y}\right)-\frac{\partial}{\partial z}\left(\frac{1}{\zeta^M}\frac{\partial \hat{H}_x}{\partial z}\right)-jk_x\frac{\partial \xi}{\partial y}\frac{\partial \hat{E}_x}{\partial z}+jk_x\frac{\partial \xi}{\partial z}\frac{\partial \hat{E}_x}{\partial y}+\upsilon^M\hat{H}_x=-\frac{\partial}{\partial z}\left(\frac{\hat{i}_{sy}}{\zeta^M}\right) \qquad \mathbf{29}$$

In equations of (26)-(29) $\zeta^M=\sigma(1-\frac{k_x^2}{k^2})$, $\zeta^E=i\omega\mu(1-\frac{k_x^2}{k^2})$, $\upsilon^M=i\omega\mu$, $\upsilon^E=\sigma$, $\xi=\left(k_x^{\,2}-k^2\right)^{-1}$ notations were used.

There are two main differences between the partial differential equations valid for the isotropic and anisotropic case: (1) opposite the isotropic situation second order mixed partial derivatives of strike directional Fourier EM field components can be found in the anisotropic case; and (2) except the coefficient of the strike directional Fourier magnetic field component all distributed parameters reflect the effect of anisotropy.

5. Finite difference formulation

In order to model the EM response of 2D inhomogeneities the conductivity structure has to be covered by a finite rectangular grid in the course of FD method. The coefficients of (22)-(25) equations are constant values within each rectangular grid element. With the use of the transmission sheet analogy any part of the grid section with distributed parameters was replaced by grid section built up lumped circuit elements [10]. The coupled partial differential equations (26)-(29) are very similar to the equations of two coupled transmission sheets and this analogy makes it possible to fulfil the interior boundary conditions, because each grid point can be regarded as a branch point of circuit. In this way a five-point difference scheme could be applied for the solution to the isotropic 2.5-D problem due to magnetic dipole source.
However, in anisotropic 2.5D problem the five point difference scheme cannot be applied, because of the mixed partial derivatives of strike directional Fourier EM field components in the partial differential equations. The second order partial derivatives with respect to *y* or *z* can be finite differenced similarly as in the case of isotropic situation, but the terms with second order mixed partial derivatives have to be transformed before they are finite differenced. For example instead of the term which is the first mixed second derivative of the strike directional Fourier magnetic field component in equation (22) it can be written as:

$$\frac{k_x}{\omega\mu}\frac{\partial}{\partial y}\left(\rho_t q_t\frac{\partial \hat{H}_x}{\partial z}\right)=\frac{k_x}{\omega\mu}\frac{\partial(\rho_t q_t)}{\partial y}\frac{\partial \hat{H}_x}{\partial z}+\frac{k_x\rho_t q_t}{\omega\mu}\frac{\partial^2 \hat{H}_x}{\partial y\partial z} \qquad \mathbf{30}$$

The linear variation of resistivity values between the center of grid element is supposed. The derivatives of EM Fourier components are approximated by their corresponding finite differences. This way both terms of the right hand side of (30) can be finite differenced. The other term of second order mixed partial derivative in (22) can be approximated similarly resulting in a nine point finite difference scheme. All terms of the right hand side in (22)-(25) are the average values of the area being lumped. After doing that instead of (22) and (23) the substituting resultant finite difference equations for the grid point containing the strike directional source term can be given in the form of:

$$\sum_{i=1}^{8}\left(\frac{\hat{E}_{ix}-\hat{E}_{0x}}{-A_i^E}+\frac{\hat{H}_{ix}-\hat{H}_{0x}}{B_i^E}\right)+C_0^E\hat{E}_{0x}=-\hat{i}_{s0x}\Delta y\Delta z \qquad \mathbf{31}$$

$$\sum_{i=1}^{8}\left(\frac{\hat{H}_{ix}-\hat{H}_{0x}}{-A_i^M}+\frac{\hat{E}_{ix}-\hat{E}_{0x}}{B_i^M}\right)+C_0^M\hat{H}_{0x}=0 \qquad \mathbf{32}$$

where *0* denotes the central grid point, *i* denotes the surrounding eight grid points in the next vicinity of the central grid point, Δy and Δz is the sum of the horizontal and vertical grid sizes in the vicinity of the central grid point, finally A_i, B_i, C_0 stand for the coefficients derived from the nine point finite difference scheme. The source terms are considered as distributed parameters without strike directional extension, making the determination of the Fourier transform of the electric source possible. The FD source term is the average value over the grid element being lumped. The electric source parallel to the dip direction is treated as Heaviside function of space in the *x=0* plane. This source term is smeared within a rectangular grid element [3], which makes it possible to take its derivative with respect *y* and *z*. The derivatives of source terms are taken across element boundaries and can be expressed in the terms of weighted delta functions. These derivatives are lumped over the line segment connecting the centre of the neighbouring grid elements and these values are referred to

the two neighbouring nodes on the element boundary. Along the edge of the grid either plane wave or terminal impedance type boundary condition can be used.
One of the most important questions in the course of FD method is the way of the solution to the linear equation system. A block tridiagonal LU decomposition (based on [11]) was suggested by [10] for the solution of the linear equation system derived by finite difference method in the isotropic situation. This method has several advantages, because it takes into account the properties of coefficient matrix, where the two co-diagonal blocks are next to the main diagonal block. For this reason instead of the inversion of the coefficient matrix (with *4mn*4mn* real elements, where m denotes the number of rows and n stands for the number of columns of the rectangular grid) this procedure reduces the problem for inverting *n* times block matrices (with *4m*4m* real elements), and the result of inversion can be used if the aim is the determination of EM fields for other type of sources or for a new source position as well. In the anisotropic case there are non-zero coefficients outside the three blocks next to each other. For this reason the block tridiagonal LU decomposition cannot be applied, however LU decomposition of band matrix seems to be a feasible solution [12].

6. Inverse Fourier transform to gain spatial solutions

The general relationship resulting in space-domain solution from the wave number can be presented as:

$$F(x,y,z) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\hat{F}(k_x,y,z)e^{jk_x x}dk_x \qquad \textbf{33}$$

For ***TE mode*** it can be proved that in broadside configuration E_x, H_y, H_z components are different from zero. Using inverse transform (33) for the vertical plane containing the source (x=0) the strike directional electric field component for any grid point:

$$E_x(0,y,z) = \frac{1}{\pi}\int_{0}^{\infty}\hat{E}_x(k_x,y,z)\,dk_x \qquad \textbf{34}$$

From equations (21) and (19) with the assumption of source free terms the Fourier-transform of non-strike directional components can be determined. For the vertical plane containing the source (x=0) the spatial magnetic components on the basis of (33):

$$H_y(0,y,z) = \frac{1}{\pi}\int_{0}^{\infty}\left[\frac{jq_n}{\omega\mu}\frac{\partial\hat{E}_x}{\partial z} - \frac{k_x\rho_n q_n}{\omega\mu}\frac{\partial\hat{H}_x}{\partial y}\right]dk_x \qquad \textbf{35}$$

$$H_z(0,y,z) = -\frac{1}{\pi}\int_{0}^{\infty}\left[\frac{jq_t}{\omega\mu}\frac{\partial\hat{E}_x}{\partial y} + \frac{k_x\rho_t q_t}{\omega\mu}\frac{\partial\hat{H}_x}{\partial z}\right]dk_x \qquad \textbf{36}$$

For ***TM mode*** in collinear configuration the other three electromagnetic field components
- E_y, E_z, H_x - are not zero and they can similarly be expressed as for TE mode. The strike directional magnetic field component can be determined as the inverse Fourier-transform of the wave number solution:

$$H_x(0,y,z) = \frac{1}{\pi}\int_{0}^{\infty}\hat{H}_x(k_x,y,z)\,dk_x \qquad \textbf{37}$$

For TM mode in collinear array one electric field component is perpendicular to the strike and the other one is vertical. Taking into account the general relationship resulting in space-domain solution from the wave number domain (33) and with the assumption of source free region the Fourier-transform of non-strike directional components we can be determine them from (17) and (14):

$$E_y(0,y,z) = \frac{1}{\pi}\int_{0}^{\infty}\left[\rho_t q_t\frac{\partial\hat{H}_x}{\partial z} - \frac{k_x\rho_t q_t}{\omega\mu}\frac{\partial\hat{E}_x}{\partial y}\right]dk_x \qquad \textbf{38}$$

$$E_z(0,y,z) = -\frac{1}{\pi}\int_0^{\infty}[\ \rho_n q_n \frac{\partial \hat{H}_x}{\partial y} + \frac{k_x \rho_n q_n}{\omega\mu}\frac{\partial \hat{E}_x}{\partial z}\ ]dk_x \qquad \textbf{39}$$

Before modelling not only the geometry of the grid, but the along-strike wave numbers -both the range and the distribution of the discrete wave numbers- have to be planned, too. In the course of planning the 1D forward modelling results are considered reference results. The spatial electromagnetic field components are determined numerically. The EM field components and their partial derivatives in (34)-(39) are approximated by second-order polynomial functions for each section of k_x (where one section is determined by three discrete k_x values). Logarithmically equidistant sampling can be suggested in the wave number domain.

7. Summary

The effect of anisotropy cannot be abandoned in the course of EM exploration. In the paper the basic formalism was presented for elongated anisotropic conductivity structures exited by electric dipole source treated as a point source with either strike or dip direction. The partial differential equations were derived and comparison was also made between the isotropic and anisotropic case. For the solution of the resulting FD linear equation system LU decomposition of band matrix is suggested, because in contrast with isotropic case 9 point FD scheme is needed in anisotropic situation. Finally the equations for the spatial solutions of EM field components are also presented and recommendation for their numerical evaluation was also given.

List of Symbols

$\vec{E}$: the electric field vector (V/m)
$\vec{H}$: the magnetic field vector (A/m)
$\vec{B}$: the magnetix flux density (Wb/m^2)
ε : the dielectric constant (F/m)
σ : the conductivity (mho/m or S/m)
ρ : resistivity (ohmm)
μ : the magnetic permeability (H/m)
r : transmitter-receiver separation, distance (m)
δ : skin depth (m)
t : time (s)
ω : the angular frequency of the EM field (Hz)
I : transmitter current (A)
$d\vec{s}$: elementary dipole length (m)
$\delta(r)$: Dirac-delta function
j : $\sqrt{-1}$
$\vec{i}$: the current density vector (A/m^2)
$\vec{i}_s$: the applied current source (A/m^2)
F : EM component (V/m, or A/m)
k_x : wave number in the strike direction (1/m)
$\hat{F}$: the Fourier-transform of the F EM component (V/m, or A/m)
k : the complex wave number (1/m)
q_n , q_t : derived dimensionless parameters
ζ^M : TM impedance (S/m)
ζ^E : TE impedance (Hm^{-1}s^{-1})
υ^M : TM admittance (Hm^{-1}s^{-1})
υ^E : TE admittance (S/m)
ξ : derived parameter (m^2)
A_i, B_i, C_0: coefficient matrix elements derived from the nine point FD scheme
Δy : the sum of the horizontal grid sizes in the vicinity of the central grid point (m)
Δz : the sum of the vertical grid sizes in the vicinity of the central grid point (m)

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REFERENCES

[1] TAKÁCS, E.: *Geophysics, Geoelectrical exploration methods I., II. (in Hungarian),* Budapest**,** Tankönyvkiadó, 1981.
[2] NABIGHIEN, M.,N.: *Electromagnetic methods in applied geophysics, Vol.2, Application, Parts A and B,* Tulsa, SEG, 1992.
[3] PETHŐ, G.: *FEM numerical modeling of 2D structures exited by electrical dipole source (in Hungarian, thesis for Candidate of Science degree),* Miskolc, 1993.
[4] PETHŐ, G.: *CSAMT Numerical Modelling for 2D Thermal EOR Monitoring*, EAPG-6th Conference and Technical Exhibition, Extended Abstracts of Papers, P547, Vienna, 1994.
[5] PETHŐ, G., KAIKKONEN, P., VANYAN, L., L.: *Numerical modelling for the effect of a 2-D seafloor trench on sea-bottom EM measurements using horizontal electric dipole sources,* Geophysica, Vol. 31., pp. 1-21, Helsinki, 1995.
[6] PETHŐ, G., KAIKKONEN, P., VANYAN, L. L.: *Frequency domain electromagnetic source polarization investigation in case of a 2-D seafloor trench.* 3rd European Environmental and Engineering Geophysics Conference, Section New Developments, Proceedings pp. 323-326, Aarhus, 1997.
[7] PETHŐ, G., FICSÓR, L.: *Distortion effect of elongated surface conductivity-inhomogeneities on CSAMT measurements.* 14th Workshop on EM induction in the Earth, Book of Abstract, p. 100, Sinaia, 1998.
[8] PETHŐ, G., FICSÓR, L.:*Source polarization effect in case of elongated surface inhomogeneities covering transition zone.* EAGE 63rd Conference & Technical exhibition, Abstract II, P050, Amsterdam, 2001.
[9] PETHŐ, G.: *FEM source effect investigation with 2.5D numerical modeling*, 15th European Meeting of Environmental and Engineering Geophysics, Near Surface Proceedings &Exhibitors'Cataloge, P59, Dublin, 2009.
[10] STOYER, C., H., GREENFIELD, R., J.: *Numerical solutions of the response of a two-dimensional earth to an oscillating magnetic dipole source*, Geophysics, 41, (1976), 519-530.
[11] SCHECHTER, S.: *Quasi-tridiagonal matrices and type-insensitive difference equations.* Quart. Appl. Math., 18, 3, pp.286-293, 1960
[12] PRESS, W., H.: *Numerical Recipes in Fortran 90,* Cambridge University Press, 1996

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BASES OF FD MODELLING FOR EM UNDERGROUND TRANSILLUMINATION WITH VERTICAL ELECTRIC DIPOLES IN 2D ANISOTROPIC CONDUCTIVITY STRUCTURES

ABSTRACT

The fundamental relationships are derived for frequency domain 2.5D situation when the transmitter is an underground vertical electric dipole source and its electromagnetic fields are observed in another borehole or in a gallery. Derivations are provided for both the isotropic and the anisotropic case. From the Fourier transforms of the Maxwell's equations coupled partial differential equations are derived, in which -besides the spatial strike directional wavenumber, the frequency and the electromagnetic rock properties- only the strike direction electromagnetic (EM) field components and their partial derivatives in the wavenumber (k_x) domain can be found. On the basis of comparison between the differential equations for the isotropic and anisotropic case it can be experienced that in contrast with the isotropic situation second order mixed partial derivatives of strike directional Fourier EM field component are in the anisotropic case. In the differential equations for the anisotropic case except the coefficient of the strike directional Fourier magnetic field component all coefficients reflect the effect of anisotropy. The equations for the spatial solutions of EM field components can be given by inverse Fourier transformation. The numerical experience gained for isotropic 2.5D TE and TM mode can be taken into account in the course of grid and spatial wavenumber domain planning in the anisotropic situation.

Keywords: anisotropy, elongated structure, frequency domain electromagnetic method, FD procedure, wavenumber domain, inverse Fourier transformation.

1. INTRODUCTION

In the course of hydrocarbon and geothermal exploration the geophysical EM methods are among the most important ones after seismic reflection survey, and the simultaneous application of the two geophysical methods is very effective. The ground-based geophysical EM methods use either natural or artificial sources [1], [2]. The attenuation of the EM fields can be described in the function of EM parameters including electrical conductivity, dielectric permittivity, magnetic permeability and frequency in general [3]. The borehole EM methods always apply transmitter(s) sending EM waves at different frequencies and receiver(s) measuring the transit time but at least wave-amplitude attenuation of the emitted EM waves [4]. The transmitters and the receivers are on the same tool body. In this paper emphasis is put on the lower frequency range where the attenuation of EM fields in homogeneous space can be characterized by the induction number which is the ratio of the transmitter-receiver separation and the skin depth [5]. In the situation presented here the source field is a vertical electric dipole source and the investigated conductivity structure is elongated. For this reason the problem to be solved is a 3D one. In order to handle the original problem more easily the

EM source is considered mathematically as a point source. This situation is called a 2.5D forward problem. In the course of EM investigation on elongated conductivity structures exited by electric dipole distinction is made between TE mode, when the electric source field is parallel to the structural strike, and TM mode, when the electric source field is perpendicular to it. In contrast with MT 2D these modes are not separate from each other they are coupled. In the case of vertical electric dipole source the derivation is very similar to those of horizontal electric dipole sources [6]. The investigation of some effects for elongated isotropic conductivity inhomogeneities exited by horizontal electric dipole source was presented by [7], [8], [9], [10], [11] and [12]. These papers dealt with the determination of EM responses over 2D isotropic conductivity structures excited by horizontal electric dipole sources and the computed EM field components were horizontal electric and magnetic field components and the impedance derived from them. In this paper emphasis is put on the EM fields determination due to vertical electric dipole source placed in 2D surroundings which can be isotropic or anisotropic as well. The effect of anisotropy is investigated by many authors and one of the best summaries is given by [13].

2. 2.5D FEM PROBLEM FOR ISOTROPIC CASE WITH VERTICAL ELECTRIC DIPOLE SOURCE

In the frequency domain electromagnetic (FEM) methods $e^{j\omega t}$ time dependent source field is assumed and let x be parallel to the structural strike direction. The basic relationships governing the electromagnetic phenomenon are the Maxwell's equations. One of them expresses that electric fields are the results from the time-varying magnetic induction:

$$rot\vec{E} = -\partial\vec{B}/\partial t \qquad \mathbf{1}$$

$\vec{i}_s = Id\vec{s}\,\delta(\vec{r}\,)$ describes a point source approximation which can be involved into the other Maxwell's equation:

$$rot\vec{H} = \vec{i} + \vec{i}_s \qquad 2$$

(2) Represents the fact that the magnetic fields are caused by conduction and source currents.

In our situation conduction currents dominate over displacement currents due to the EM fields of low frequencies applied. For this reason the effect of displacement currents in (2) was not taken into account.

In the case of magnetic source [14] recommended that the solution of the original 3D problem could be replaced by the series of solutions of 2D problem in the k_x wavenumber domain. This procedure assumes that the Maxwell's equations (1) and (2) are Fourier-transformed. This transformation can be defined as:

$$\hat{F}(k_x, y, z) = \int_{-\infty}^{\infty} F(x, y, z) e^{-jk_x x} dx \quad \mathbf{3}$$

In the course of the transformation the Fourier transform of the partial derivative of the EM field components with respect x is also required. It can be proved that the partial derivative of any EM field components with respect x is proportional to the Fourier transform of the component itself (see e.g. [6]).

Taking into account the harmonic time dependence of EM fields, we can rewrite the Fourier transforms of (1) Maxwell's equation in the following form:

$$\frac{\partial \hat{E}_z}{\partial y} - \frac{\partial \hat{E}_y}{\partial z} = -j\omega\mu \hat{H}_x \quad \mathbf{4}$$

$$\frac{\partial \hat{E}_x}{\partial z} - jk_x \hat{E}_z = -j\omega\mu \hat{H}_y \quad \mathbf{5}$$

$$jk_x \hat{E}_y - \frac{\partial \hat{E}_x}{\partial y} = -j\omega\mu \hat{H}_z \quad \mathbf{6}$$

If we assume that only vertical electric dipole source is applied, the Fourier transforms of (2) can be written as:

$$\frac{\partial \hat{H}_z}{\partial y} - \frac{\partial \hat{H}_y}{\partial z} = \sigma \hat{E}_x \quad \mathbf{7}$$

$$\frac{\partial \hat{H}_x}{\partial z} - jk_x \hat{H}_z = \sigma \hat{E}_y \quad \mathbf{8}$$

$$jk_x \hat{H}_y - \frac{\partial \hat{H}_x}{\partial y} = \sigma \hat{E}_z + \hat{i}_{sz} \quad \mathbf{9}$$

The goal of the next part is to provide partial differential equation systems with equations containing only strike direction EM field components in the spatial wavenumber (k_x) domain. The first equation (the pure one) belonging to the TM mode is (4). From equations (5) and (9) $\hat{H}_y$ can be determined and taking them to be equal we receive:

$$\sigma(1 - k_x^2 / k^2)\hat{E}_z = -\frac{\partial \hat{H}_x}{\partial y} - \frac{k_x}{\omega\mu} \frac{\partial \hat{E}_x}{\partial z} - \hat{i}_{sz} \quad \mathbf{10}$$

Where k denotes the complex wavenumber and $k^2 = -j\omega\mu\sigma$

From equations (6) and (8) $\hat{H}_z$ can be determined and taking them to be equal we obtain:

$$-\sigma(1 - k_x^2 / k^2)\hat{E}_y = -\frac{\partial \hat{H}_x}{\partial z} + \frac{k_x}{\omega\mu} \frac{\partial \hat{E}_x}{\partial y} \quad \mathbf{11}$$

The other group of equations represents the TE mode. Following the 2D MT analogy one equation belonging to this mode is (7).
The non-strike directional magnetic field components in the wavenumber domain can be determined similarly as the non-strike directional electric field components were done for the TM mode. From equations (6) and (8) $\hat{E}_y$ can be expressed and taking them to be equal we have:

$$-j\omega\mu(1-k_x^{\,2}/k^2)\hat{H}_z = -\frac{\partial \hat{E}_x}{\partial y} + \frac{jk_x}{\sigma}\frac{\partial \hat{H}_x}{\partial z} \qquad \mathbf{12}$$

From equations (5) and (9) $\hat{E}_z$ can be expressed and taking them to be equal we get:

$$j\omega\mu(1-k_x^{\,2}/k^{\,2})\hat{H}_y = -\frac{\partial \hat{E}_x}{\partial z} - \frac{jk_x}{\sigma}\frac{\partial \hat{H}_x}{\partial y} - \frac{jk_x}{\sigma}\hat{i}_{sz} \qquad \mathbf{13}$$

Let us introduce $\zeta^E = i\omega\mu\,(1-\frac{k_x^2}{k^2})\,;\, \xi = \left(k_x^{\,2}-k^2\right)^{-1};\, \upsilon^E = \sigma$, and substitute the value of $\hat{H}_y$ and $\hat{H}_z$ -from (13) and (12), respectively - into (7):

$$-\frac{\partial}{\partial y}\left(\frac{1}{\zeta^E}\frac{\partial \hat{E}_x}{\partial y}\right) - \frac{\partial}{\partial z}\left(\frac{1}{\zeta^E}\frac{\partial \hat{E}_x}{\partial z}\right) + jk_x\frac{\partial \xi}{\partial y}\frac{\partial \hat{H}_x}{\partial z} - jk_x\frac{\partial \xi}{\partial z}\frac{\partial \hat{H}_x}{\partial y} + \upsilon^E\hat{E}_x = jk_x\frac{\partial}{\partial z}\left(\xi\,\hat{i}_{sz}\right) \qquad \mathbf{14}$$

Let us introduce $\zeta^M = \sigma\,(1-\frac{k_x^2}{k^2})\,;\, \upsilon^M = i\omega\mu$, and substitute the value of $\hat{E}_y$ and $\hat{E}_z$ - from (11) and (10), respectively- into (4):

$$-\frac{\partial}{\partial y}\left(\frac{1}{\zeta^M}\frac{\partial \hat{H}_x}{\partial y}\right) - \frac{\partial}{\partial z}\left(\frac{1}{\zeta^M}\frac{\partial \hat{H}_x}{\partial z}\right) - jk_x\frac{\partial \xi}{\partial y}\frac{\partial \hat{E}_x}{\partial z} + jk_x\frac{\partial \xi}{\partial z}\frac{\partial \hat{E}_x}{\partial y} + \upsilon^M\hat{H}_x = \frac{\partial}{\partial y}\left(\frac{\hat{i}_{sz}}{\zeta^M}\right) \qquad \mathbf{15}$$

In contrast with 2D MT, the TE and TM mode cannot be treated separately, and the partial differential equation system consisting of (14) and (15) has to be solved. In the isotropic situation -governed by (14) and (15) - second order mixed partial derivatives of the strike directional Fourier EM field components cannot be found. Independent of the type of source there is no pure TM and TE mode, because the two polarizations are coupled for each source type (direction) case. Still this statement in the course of 2.5D modeling the case of electric dipole source of structural strike direction is called TE mode, and when the source term is perpendicular to the structural strike it is referred as TM mode [6], [7]. The reason of this nomination is the following: in the case of electric dipole source parallel to the structural strike the current flow is mainly parallel to the strike, and for the situation when the source term is perpendicular to the structural strike there is a horizontal magnetic field component which coincides with the structural strike just like in 2D MT TM mode. There is a similarity between differential equations for 2.5D TM mode and the ones involving vertical dipole source in a 2D conductivity surrounding: in both cases the inhomogeneous terms of the equations are the partial derivatives of the applied current source with respect *y* or *z*, but in a changed order.

3. 2.5D FEM PROBLEM FOR ANISOTROPIC CASE WITH VERTICAL ELECTRIC DIPOLE SOURCE

The conductivity determining the relationship between the current density vector and the electric field vector is a scalar quantity only for an isotropic situation, in general case it is a tensor. The conductivity tensor has a simple form if two of the orthogonal coordinate directions are assumed to lie in the direction of maximum conductivity and the third one into the minimum conductivity value. In the case of layering the greatest resistivity can be observed perpendicularly to the layering. The same σ_t conductivity can be usually proposed in the *x* and *y* direction and σ_n value denotes the conductivity value in the *z* direction. The presence of anisotropy does not influence the Fourier transforms of (1) Maxwell's equation, these component equations (4)-(6) remain unchanged. Taking into account the anisotropy in the Fourier transforms of (2) instead of (7)-(9) we can write:

$$\frac{\partial \hat{H}_z}{\partial y} - \frac{\partial \hat{H}_y}{\partial z} = \sigma_t \hat{E}_x \qquad \textbf{16}$$

$$\frac{\partial \hat{H}_x}{\partial z} - jk_x \hat{H}_z = \sigma_t \hat{E}_y \qquad \textbf{17}$$

$$jk_x \hat{H}_y - \frac{\partial \hat{H}_x}{\partial y} = \sigma_n \hat{E}_z + \hat{i}_{sz} \qquad \textbf{18}$$

The aim of the next derivation is the same as it was in isotropic case: to yield a partial differential equation system containing only strike direction EM field components and their partial derivatives with respect space in the wavenumber (k_x) domain besides distributed parameters and frequency. From equations (5) and (18) $\hat{H}_y$ can be determined and taking them to be equal we get:

$$\sigma_n (1 - k_x^2 / k_n^2) \hat{E}_z = -\frac{\partial \hat{H}_x}{\partial y} - \frac{k_x}{\omega\mu} \frac{\partial \hat{E}_x}{\partial z} - \hat{i}_{sz} \qquad \textbf{19}$$

If we introduce

$$q_n = 1 / (1 - k_x^2 / k_n^2) \qquad \textbf{20}$$

and we use the resistivity instead of conductivity in (19)

$$\hat{E}_z = -\rho_n q_n \frac{\partial \hat{H}_x}{\partial y} - \frac{k_x \rho_n q_n}{\omega\mu} \frac{\partial \hat{E}_x}{\partial z} - \rho_n q_n \hat{i}_{sz} \qquad \textbf{21}$$

From equations (6) and (17) $\hat{H}_z$ can be determined and taking them to be equal we get:

$$-\sigma_t(1-k_x{}^2 / k_t{}^2)\hat{E}_y = -\frac{\partial \hat{H}_x}{\partial z} + \frac{k_x}{\omega\mu}\frac{\partial \hat{E}_x}{\partial y} \quad \textbf{22}$$

Introducing

$$q_t = 1 / (1-k_x{}^2 / k_t^2) \quad \textbf{23}$$

and using the resistivity instead of conductivity from (22)

$$\hat{E}_y = \rho_t q_t \frac{\partial \hat{H}_x}{\partial z} - \frac{k_x \rho_t q_t}{\omega\mu}\frac{\partial \hat{E}_x}{\partial y} \quad \textbf{24}$$

The vertical magnetic field components in the wavenumber domain can be determined similarly. From equations (6) and (17) $\hat{E}_y$ can be expressed and taking them to be equal we get:

$$-j\omega\mu(1-k_x{}^2 / k_t{}^2)\hat{H}_z = -\frac{\partial \hat{E}_x}{\partial y} + \frac{jk_x}{\sigma_t}\frac{\partial \hat{H}_x}{\partial z} \quad \textbf{25}$$

From this equation $\hat{H}_z$ can be expressed as:

$$\hat{H}_z = -\frac{jq_t}{\omega\mu}\frac{\partial \hat{E}_x}{\partial y} - \frac{k_x \rho_t q_t}{\omega\mu}\frac{\partial \hat{H}_x}{\partial z} \quad \textbf{26}$$

The horizontal magnetic field component perpendicular to the strike can be determined in the following way: from equations (5) and (18) $\hat{E}_z$ can be expressed and taking them to be equal we obtain:

$$j\omega\mu(1-k_x{}^2 / k_n{}^2)\hat{H}_y = -\frac{\partial \hat{E}_x}{\partial z} - \frac{jk_x}{\sigma_n}\frac{\partial \hat{H}_x}{\partial y} - \frac{jk_x}{\sigma_n}\hat{i}_{sz} \quad \textbf{27}$$

From this equation $\hat{H}_y$ can be expressed as:

$$\hat{H}_y = \frac{jq_n}{\omega\mu}\frac{\partial \hat{E}_x}{\partial z} - \frac{k_x \rho_n q_n}{\omega\mu}\frac{\partial \hat{H}_x}{\partial y} - \frac{k_x \rho_n q_n}{\omega\mu}\hat{i}_{sz} \quad \textbf{28}$$

In the knowledge of the non-strike directional EM field components in the wavenumber domain we can substitute them into the "pure" TE and TM components equations: substituting (26) and (28) into (16) we receive (29). The other equation will be (30) which can be derived by the substitution of (21) and (24) into (6):

$$\frac{j}{\omega\mu}\frac{\partial}{\partial y}\left(q_t \frac{\partial \hat{E}_x}{\partial y}\right) + \frac{k_x}{\omega\mu}\frac{\partial}{\partial y}\left(\rho_t q_t \frac{\partial \hat{H}_x}{\partial z}\right) + \frac{j}{\omega\mu}\frac{\partial}{\partial z}\left(q_n \frac{\partial \hat{E}_x}{\partial z}\right) - \frac{k_x}{\omega\mu}\frac{\partial}{\partial z}\left(\rho_n q_n \frac{\partial \hat{H}_x}{\partial y}\right) + \sigma_t \hat{E}_x = \frac{k_x}{\omega\mu}\frac{\partial}{\partial z}\left(\rho_n q_n \hat{i}_{sz}\right) \quad \textbf{29}$$

$$\frac{\partial}{\partial y}\left(\rho_n q_n \frac{\partial \hat{H}_x}{\partial y}\right)+\frac{k_x}{\omega\mu}\frac{\partial}{\partial y}\left(\rho_n q_n \frac{\partial \hat{E}_x}{\partial z}\right)+\frac{\partial}{\partial z}\left(\rho_t q_t \frac{\partial \hat{H}_x}{\partial z}\right)-\frac{k_x}{\omega\mu}\frac{\partial}{\partial z}\left(\rho_t q_t \frac{\partial \hat{E}_x}{\partial y}\right)-j\omega\mu\hat{H}_x=-\frac{\partial}{\partial y}\left(\rho_n q_n \hat{i}_{sz}\right)$$

30

Except the coefficient of the strike directional Fourier magnetic field component in (30) all coefficients reflect the effect of anisotropy, even the inhomogeneous terms do in equations of (29) and (30). In contrast with the isotropic situation second order mixed partial derivatives of the strike directional Fourier EM field components are in the anisotropic case. In order to obtain the finite difference form of (29) and (30) additional transformations are suggested. Instead of the term for the first mixed second order partial derivative of the strike directional Fourier magnetic field components in (29) it can be written as:

$$\frac{k_x}{\omega\mu}\frac{\partial}{\partial y}\left(\rho_t q_t \frac{\partial \hat{H}_x}{\partial z}\right)=\frac{k_x}{\omega\mu}\frac{\partial(\rho_t q_t)}{\partial y}\frac{\partial \hat{H}_x}{\partial z}+\frac{k_x\rho_t q_t}{\omega\mu}\frac{\partial^2 \hat{H}_x}{\partial y\partial z} \quad \textbf{31}$$

Similarly, for the second mixed second order partial derivative of the strike directional Fourier magnetic field components in (29):

$$\frac{k_x}{\omega\mu}\frac{\partial}{\partial z}\left(\rho_n q_n \frac{\partial \hat{H}_x}{\partial y}\right)=\frac{k_x}{\omega\mu}\frac{\partial(\rho_n q_n)}{\partial z}\frac{\partial \hat{H}_x}{\partial y}+\frac{k_x\rho_n q_n}{\omega\mu}\frac{\partial^2 \hat{H}_x}{\partial z\partial y} \quad \textbf{32}$$

Finally, instead of the mixed second order partial derivatives of the strike directional Fourier electric field component in (30):

$$\frac{k_x}{\omega\mu}\frac{\partial}{\partial y}\left(\rho_n q_n \frac{\partial \hat{E}_x}{\partial z}\right)=\frac{k_x}{\omega\mu}\frac{\partial(\rho_n q_n)}{\partial y}\frac{\partial \hat{E}_x}{\partial z}+\frac{k_x\rho_n q_n}{\omega\mu}\frac{\partial^2 \hat{E}_x}{\partial y\partial z} \quad \textbf{33}$$

$$\frac{k_x}{\omega\mu}\frac{\partial}{\partial z}\left(\rho_t q_t \frac{\partial \hat{E}_x}{\partial y}\right)=\frac{k_x}{\omega\mu}\frac{\partial(\rho_t q_t)}{\partial z}\frac{\partial \hat{E}_x}{\partial y}+\frac{k_x\rho_t q_t}{\omega\mu}\frac{\partial^2 \hat{E}_x}{\partial z\partial y} \quad \textbf{34}$$

4. FINITE DIFFERENCE FORMULATION

In order to forward model the EM response due to a 2D structure with finite difference (FD) method, the conductivity inhomogeneities and the homogeneous host have to be covered by a finite rectangular grid. The grid geometry has to be also fit to the conductivity boundaries, the positions of the source and the receivers. The physical parameters of (29)-(30) equations are constant within each rectangular grid element and these parameters are considered as distributed parameters. For the isotropic case there must be two input conductivity maps, one for the tangential, and the other one for the normal conductivity distribution. With the use of the transmission sheet analogy any part of the grid section with distributed parameters can be replaced by a grid section built up lumped circuit elements [14]. The coupled partial differential equations (29)-(30) are very similar to the equations of two coupled transmission sheets and this analogy makes it possible to fulfill the interior boundary conditions, because

each grid point can be regarded as a branch point of circuit. In this way a five-point difference scheme could be applied for the FD forward modeling to the isotropic 2.5-D problem due to a dipole source. The second order partial derivatives with respect to *y* or *z* can be finite differenced similarly as in the case of isotropic situation [6] and [8]. However, for the anisotropic 2.5D problem -due to the mixed partial derivatives of strike directional Fourier EM field components in the partial differential equations of (29)-(30)- a nine point finite difference scheme can be suggested.
The linear variation of resistivity values between the centers of grid element is supposed. The derivatives of the EM Fourier components are approximated by their corresponding finite differences. In this way both terms of the right hand side of (31)-(34) can be finite differenced. The term of the second order mixed partial derivative can be approximated by finite differences if we take into account the field values in eight grid points surrounding the central node. All terms of the right hand side in (29)-(30) are the average values of the area being lumped. The vertical electric source is treated as a Heaviside function of space in the *x=0* plane. This source term is smeared within a rectangular grid element, like the horizontal electric dipole source perpendicular to the strike [6], which makes it possible to take its derivative with respect *y* and *z*. The derivatives of the source terms are taken across element boundaries and can be expressed in the terms of weighted delta functions. These derivatives are lumped over the line segment connecting the centre of the neighboring grid elements and these average values are referred to the two neighboring nodes on the element boundary. Because of this source treatment there are four grid points with non-zero source term. Instead of (29)-(30) the substituting resultant finite difference equations for the grid point containing the vertical dipole source term can be expressed in the following form:

$$\sum_{i=1}^{8}\left(\frac{\hat{E}_{ix}-\hat{E}_{0x}}{-A_i^E}+\frac{\hat{H}_{ix}-\hat{H}_{0x}}{B_i^E}\right)+C_0^E\hat{E}_{0x}=\hat{S}_{0z}^E \qquad \mathbf{35}$$

$$\sum_{i=1}^{8}\left(\frac{\hat{H}_{ix}-\hat{H}_{0x}}{-A_i^M}+\frac{\hat{E}_{ix}-\hat{E}_{0x}}{B_i^M}\right)+C_0^M\hat{H}_{0x}=\hat{S}_{0z}^M \qquad \mathbf{36}$$

here *0* stands for the central grid point and *i* denotes the surrounding eight grid points in the vicinity of the central node. A_i, B_i, C_0 stand for the coefficients derived from the nine point finite difference scheme and $\hat{S}_{0x}$ denote the finite difference source terms for the vertical dipole source in the wavenumber domain. Along the edge of the grid either plane wave or terminal impedance type boundary condition can be used. In the course of our isotropic modeling the terminal impedance type boundary condition was preferred.
The way of the solution to the linear equation system is one of the most important questions in the course of FD method. The block tridiagonal LU decomposition suggested by [14] and [15] for the solution of the linear equation system can be used only for the isotropic situation. This procedure takes into account the properties of coefficient matrix. In isotropic case only the two co-diagonal and the main diagonal blocks are non-zero matrices (with *4m*4m* real elements). For this reason instead of the inversion of the coefficient matrix (with *4mn*4mn* real elements, where *m* denotes the number of rows and *n* stands for the number of columns of the grid) the *n* times inversion of block matrices was done. In the anisotropic case there are non-zero matrix coefficients outside the three central blocks next to each other. It means that the block tridiagonal LU decomposition cannot be applied. At the same time LU decomposition of band matrix seems to be one of the promising solutions [16]. For the solution of banded great linear system parallel processing can be suggested.

5. INVERSE FOURIER TRANSFORMS TO OBTAIN SPATIAL SOLUTIONS

The general relationship resulting in space-domain solution from the wavenumber can be given as:

$$F(x,y,z)=\frac{1}{2\pi}\int_{-\infty}^{\infty}\hat{F}(k_x,y,z)e^{jk_x x}dk_x \qquad \mathbf{37}$$

For the plain containing the source and the vertical borehole (or any underground observation point) this relationship -where y_R denotes the y co-ordinate of the receiver- can be written as:

$$F(0,y_R,z)=\frac{1}{2\pi}\int_{-\infty}^{\infty}\hat{F}(k_x,y_R,z)dk_x \qquad \mathbf{38}$$

This equation can be directly applied to the spatial determination of strike-directional magnetic field component, because in the spatial wavenumber domain this component was given for any node of the grid as the solution of the linear equation system:

$$H_x(0,y_R,z)=\frac{1}{\pi}\int_{0}^{\infty}\hat{H}_x(k_x,y_R,z)\,dk_x \qquad \mathbf{39}$$

In the plane containing the source and perpendicular to the strike E_y and E_z can be measured, too. These components can be expressed from the component Maxwell's equations (4)-(9) with the assumption of no source term. As it can be seen the equation for the strike-directional magnetic field component does not contain conductivity anisotropy dependence as opposed to the non strike-directional electric field components. For the isotropic case let us introduce $q_R=1/(1-k_x^{\,2}/k_R^{\,2})$, here R refers to the actual receiver position and taking into account (11):

$$E_y(0,y_R,z)=\frac{1}{\pi}\int_{0}^{\infty}[\;\rho_R q_R\frac{\partial\hat{H}_x}{\partial z}-\frac{k_x\rho_R q_R}{\omega\mu}\frac{\partial\hat{E}_x}{\partial y}\;]dk_x \qquad \mathbf{40}$$

In a vertical borehole if there is no conductivity anisotropy in the receiver site (R), then the vertical electric field can be expressed from (10):

$$E_z(0,y_R,z)=-\frac{1}{\pi}\int_{0}^{\infty}[\;\rho_R q_R\frac{\partial\hat{H}_x}{\partial y}+\frac{k_x\rho_R q_R}{\omega\mu}\frac{\partial\hat{E}_x}{\partial z}\;]dk_x \qquad \mathbf{41}$$

Similar equations can be derived for these two EM components if the receiver site can be characterized by conductivity anisotropy. These EM components can be expressed from the component Maxwell's equations (4)-(6) and (16)-(18) with the assumption of no inhomogeneous term in (18):

$$E_y(0,y_R,z)=\frac{1}{\pi}\int_{0}^{\infty}[\;\rho_{tR} q_{tR}\frac{\partial\hat{H}_x}{\partial z}-\frac{k_x\rho_{tR} q_{tR}}{\omega\mu}\frac{\partial\hat{E}_x}{\partial y}\;]dk_x \qquad \mathbf{42}$$

and that of the source term free version of (21):

$$E_z(0,y_R,z) = -\frac{1}{\pi}\int_0^\infty [\ \rho_{nR} q_{nR} \frac{\partial \hat{H}_x}{\partial y} + \frac{k_x \rho_{nR} q_{nR}}{\omega\mu} \frac{\partial \hat{E}_x}{\partial z}\ ] dk_x \qquad \mathbf{43}$$

It can be seen that besides the partial derivatives of the strike-directional EM field components in spatial wavenumber domain, the electric field component perpendicular to the strike depends on the lateral, while the vertical electric field component depends on the normal conductivity value.

6. SOME ASPECTS OF GRID GEOMETRY AND SPATIAL WAVENUMBER PLANNING

Depending on the problem to be solved different conductivity distribution, transmitter-receiver arrays and measuring frequencies are assumed. The grid geometry with the conductivity distribution has to reflect the 2D conductivity structure to be investigated, and in the case of anisotropic case two conductivity maps will have to be provided for the same grid geometry within the input data set. The problems solved by TE and TM mode FEM soundings resulted in some experience [6]-[12], which is worth taking into account in anisotropic situation as well [17]. The computation of frequency sounding curves requires changing grid geometry in the course of FD modeling. For grid geometry planning 1D forward modeling results were considered as references.

Due to the stronger attenuation of high frequency EM fields grid with finer meshes covering smaller area of the investigated 2D structure can be applied. Because the attenuation depends on the conductivity as well, for this reason besides the highest frequency it is the greatest tangential conductivity of the section which determines the element of the finest grid geometry. It cannot be greater than *p/10*, where *p* denotes the skin depth for this region. The greatest grid area (with courser grid elements) can be planned by the value of the greatest EM wavelength. The distance between the inhomogeneity and the edge of the grid both in horizontal and vertical sense (downward) has to be at least 2-3 times greater than the skin depth. In the case of ground EM measurements (or shallow depth EM transillumination) the vertical grid extension for air has to exceed 1.5 times the transmitter-receiver distance. At least in the vicinity of the source and the receiver the use of equidistance grid sizes are suggested, and if it possible this grid geometry can be recommended between the whole transmitter-receiver separation as well. Outside from the inhomogeneity gradual increase of grid elements with a factor of 1,5 can be accepted. The 1D reference comparison can be also used to guess the frequency at which we change the grid to compute frequency sounding curves. It is obvious that the 2.5D EM responses determined at this frequency with the two grids must be the same within accuracy better than that of the measurement.

It follows from the essence of the procedure that the range and the distribution of the along-strike wavenumbers have to be planned, too. Instead of the upper limit of the integrals of (39)-(43) it is sufficient to compute till a k_{xmax} value. We experienced that this maximum value depends on the minimal transmitter-receiver range, the highest frequency and the greatest conductivity in the region to be investigated. After $k_x=0$ value - which corresponds to the EM plane wave case- logarithmically equidistant sampling can be suggested in the wavenumber domain. The number of along-strike wavenumbers for which the linear equation system has to be solved has to exceed 21. The inverse Fourier transforms are made numerically: the EM field components and their partial derivatives in (39)-(43) are approximated by second-order polynomial functions for each section of k_x (where one section is determined by three discrete k_x values, and the third k_x value of each section is the first k_x value of the next section).

7. CONCLUSIONS

It is well known that the effect of anisotropy has to be taken into account in the course of the geoelectric and electromagnetic exploration. In this paper the basic formalism was derived for 2D isotropic and anisotropic conductivity structures exited by vertical electric dipole source treated as a point source. The partial differential equations were derived for both cases in the wavenumber domain and a comparison was also made between the partial differential equation systems. For the solution of the resulting FD banded linear equation system LU decomposition of band matrix is suggested, because in contrast with isotropic case 9 point FD scheme is required in the anisotropic situation. The experience gained for isotropic 2.5D TE and TM mode can be taken into account in the course of the grid and the spatial wavenumber domain planning for the anisotropic case. To obtain the spatial solutions of EM field components the inverse Fourier equations were also presented. Some recommendations for their numerical evaluation of inverse Fourier transforms, for the planning of spatial wavenumber sampling and some aspects of grid geometry planning were also provided.

NOMENCLATURE

A_i, B_i, C_0: coefficient matrix elements derived from the nine point FD scheme
$\vec{B}$: magnetic flux density (Wb/m^2)
$d\vec{s}$: elementary dipole length (m)
$\vec{E}$: electric field vector (V/m)
F : EM component (V/m, or A/m)
$\hat{F}$: Fourier-transform of the F EM component (V/m, or A/m)
$\vec{H}$: magnetic field vector (A/m)
I : transmitter current (A)
$\vec{i}$: current density vector (A/m^2)
$\vec{i}_s$: applied current source (A/m^2)
j : $\sqrt{-1}$
k : complex wavenumber (1/m)
k_x: wavenumber in the strike direction (1/m)
p: skin depth (m)
q_n : derived dimensionless parameters (subscript *n* refers to normal)
q_t : derived dimensionless parameters (subscript *t* refers to tangential)
r: transmitter-receiver separation, distance (m)
t : time (s)
$\delta(r)$: Dirac-delta function
ε : dielectric constant (F/m)
ζ^E : TE impedance (Hm^{-1}s^{-1})
ζ^M : TM impedance (S/m)
μ : magnetic permeability (H/m)
υ^E : TE admittance (S/m)

υ^M : TM admittance ($Hm^{-1}s^{-1}$)
ξ : derived parameter (m^2)
ρ : resistivity (ohmm)
σ : conductivity (mho/m or S/m)
ω : angular frequency of the EM field (Hz)

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REFERENCES

1. **Takács, E.:** "Geophysics, Geoelectrical exploration methods I., II." (in Hungarian), Budapest, Tankönyvkiadó, 1981.
2. **Nabighien, M. N.:** "Electromagnetic methods in applied geophysics." Vol.2, Application, Parts A and B, Tulsa, SEG, 1992.
3. **Wait, J. R.:** "Geo-Electromagnetism." Academic Press, Inc., 1982.
4. **Serra, O., Serra, L.:** "Well logging data acquisition and applications." Serralog, 2004.
5. **Takács, E.:** "In-mine frequency sounding with a buried grounded dipole source." Geophysical Transactions,Vol.34., No.4., 1988, 343-359.
6. **Pethő, G.:** "FEM numerical modeling of 2D structures exited by electrical dipole source." (in Hungarian, thesis for Candidate of Science degree), Miskolc, 1993.
7. **Pethő, G.:** "CSAMT Numerical Modelling for 2D Thermal EOR Monitoring.", EAPG-6th Conference and Technical Exhibition Extended Abstracts of Papers, P547, Vienna, 1994.
8. **Pethő, G., Kaikkonen, P., Vanyan, L., L.:** "Numerical modelling for the effect of a 2-D seafloor trench on sea-bottom EM measurements using horizontal electric dipole sources." Geophysica, Vol. 31., 1995, 1-21.
9. **Pethő, G., Kaikkonen, P., Vanyan, L., L.:** "Frequency domain electromagnetic source polarization investigation in case of a 2-D seafloor trench." Aarhus, 3rd European Environmental and Engineering Geophysics Conference, Section New Developments, 1997, 323-326.
10. **Pethő, G., Ficsór, L.:** "Distortion effect of elongated surface conductivity-inhomogeneities on CSAMT measurements." 14th Workshop on EM induction in the Earth, Book of Abstract, p. 100, Sinaia, 1998.
11. **Pethő, G., Ficsór, L.:** "Source polarization effect in case of elongated surface inhomogeneities covering transition zone." Amsterdam, EAGE 63rd Conference & Technical exhibition, Abstract II, P050, 2001.
12. **Pethő, G.:** "FEM source effect investigation with 2.5D numerical modeling." 15th European Meeting of Environmental and Engineering Geophysics, Dublin, Near Surface Proceedings &Exhibitors'Cataloge, P59, 2009.
13. **Negi, J., G., Saraf, P.,D.:** "Anisotropy in geoelectromagnetism." Methods in geochemistry and geophysics, 28., Elsevier, 1989.

14. **Stoyer, C., H., Greenfield, R., J.:** “Numerical solutions of the response of a two-dimensional earth to an oscillating magnetic dipole source.” Geophysics, 41, 1976, 519-530.
15. **Schechter, S.:** “Quasi-tridiagonal matrices and type-insensitive difference equations.” Quart. Appl. Math., 18, 3, 1960, 286-293.
16. **Press, W., H.:** “Numerical Recipes in Fortran 90.” Cambridge University Press, 1996.
17. **Pethő, G.:** “Differential equations of FEM using electric dipole source for elongated structures with conductivity anisotropy.” Kutatás és Innováció a Magyar Geotermiában, Miskolc, 2012, 70-86.

DIFFERENTIAL EQUATIONS AND COMPARISON OF FEM TE AND TM MODE WITH CONDUCTIVITY ANISOTROPY

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Abstract. Assuming conductivity anisotropy the basic relationships are derived for frequency domain electromagnetic (FEM) TE (E polarization) and TM mode (H polarization) for the 2D (two-dimensional) and the 2.5D situations. In the course of the anisotropy investigation presented here the principal directions of conductivity tensor are parallel to the main structural strike directions of the elongated conductivity structure. For the anisotropic case of 2D MT(magnetotellurics) it is proved that independent of the angle of the incident plane wave the EM field can be decomposed for two polarizations, just like in the case of 2D MT isotropic situation. For this reason the two polarizations can be treated mathematically separately. In the 2D MT TE (E polarization) mode partial differential equation the strike directional field component is the unknown and the strike directional conductivity value controls the effect of anisotropy. In the 2D MT TM (H polarization) mode the strike directional magnetic field component is the unknown and the conductivities in dip and the vertical direction control the effect of anisotropy. The 2.5D anisotropic FEM problem is solved in the strike directional wave number domain and from the Fourier transforms of the Maxwell's equations coupled partial differential equations are derived for the two polarizations. The unknowns of the differential equations are the Fourier transforms of the strike directional electric and magnetic field components. Due to the differential equation systems the two modes are coupled and the anisotropy effect is controlled by the three principal values of conductivity tensor. Approaching the theoretical far field zone similarity can be observed between the controlled and the plane wave source methods in the point of view of anisotropy effect as well.

Keywords: conductivity anisotropy, frequency domain electromagnetics, plane wave source, controlled source, E polarization, H polarization, wave number domain.

1. Introduction

The geophysical electromagnetic methods apply either natural or artificial source(s) [1, 2]. If the investigated conductivity structure is elongated and the source field is due to EM plane waves, just like in the magnetotellurics, we encounter with a pure 2D MT problem. Generally, the conductivity structures are anisotropic 3D ones and the 2D assumption is a theoretical approach. However, we frequently encounter with partially elongated conductivity structures, which are far from being 1D, they are practically 3D, but they can be approximated by 2D structures at least in the middle 1/3 part of the structure. For isotropic 2D MT it was proved ([1, 2]) that the E- and H polarization can be treated separately [1, 2]. If the source is an electric dipole source, the problem to be solved is a 3D one. There is a special situation for a 2D structure when it is exited by an electric dipole which can be approximated mathematically as a point source. If we encounter a 2D model which is exited by a point source the situation is called as a 2.5D problem. Just like in MT in the course of EM investigation on elongated conductivity structures exited by electric dipole source special emphasis has to be put on the two modes. Distinction is made between TE mode, when the electric source is parallel to the structural strike, and TM mode, when the electric source is perpendicular to it. Assuming elongated isotropic conductivity inhomogeneities exited by electric dipole source the partial differential equations were presented by Pethő [3] and besides the investigation of some effects different numerical model studies were also carried out [4], [5], [6], [7], [8], [9]. For anisotropic case differential equations were provided in [10] and [11], however, difference was made only between the conductivity values parallel to the horizontal layering and perpendicular to it. In this paper the principle directions of the conductivity tensor are parallel to the main structural direction of the 2D structure. The partial differential equations for the 2D situation with conductivity anisotropy are derived. This situation practically corresponds to MT (Magnetotellurics) and VLF (Very Low Frequency). The latter method was also applied in the CRITICEL project [12], and it is obvious that the geological structures investigated there were rather anisotropic than isotropic ones. The type of the conductivity anisotropy assumed here is far from being the general one, however, it may occur that one geoelectric domain in the inhomogeneous half-space can be characterized by this kind of anisotropy and this strongly anisotropic domain is embedded into homogeneous isotropic (layered) half-space or 2D isotropic host. For the 2.5D situation with conductivity anisotropy partial differential equation systems are derived. The similarities and differences between the two cases (2D and 2.5D) will be also provided putting emphasis on the effect of anisotropy.

2. 2D MT problem for anisotropic case

In geophysical EM exploration the basic relationships governing the electromagnetic phenomenon are the Maxwell's equations with the constitutive equations. One Maxwell's equation represents the physical law that electric fields result from time-varying magnetic induction:

$$rot\vec{E} = -\partial\vec{B}/\partial t \qquad \mathbf{1}$$

The next Maxwell's equation expresses the fact that the magnetic fields are caused by displacement and conduction current flows. In geophysical practice conduction current is dominant over displacement current (high-frequency georadar can be regarded as an exception), for this reason displacement current can be neglected:

$$rot\vec{H} = \vec{i} = [\sigma]\vec{E} \qquad \mathbf{2}$$

In order to have relationship between electromagnetic fields and subsurface conductivity the introduction of Ohm's law is required. Generally, the conductivity determining the relationship between the current density vector and the electric field vector must be a tensor (it is a scalar quantity only for an isotropic case). The conductivity tensor has a simple form if two of the orthogonal coordinate directions are selected to lie in the direction of maximum conductivity and the third one into the minimum conductivity. If the principal directions of the conductivity tensors coincide with the strike, dip and vertical directions the off-diagonal terms are zero based upon [13, 14]. In this point we remark that the anisotropy discussed in this paper is not a general treatment, we consider the situation when the principle directions of the conductivity tensor are parallel to the main structural directions of the 2D structure. We assume σ_x conductivity in the strike direction of *x*, σ_y conductivity along *y*, and σ_z conductivity value in the *z* direction.

$$\sigma = \begin{vmatrix} \sigma_{xx} & 0 & 0 \\ 0 & \sigma_{yy} & 0 \\ 0 & 0 & \sigma_{zz} \end{vmatrix} = \begin{vmatrix} \sigma_x & 0 & 0 \\ 0 & \sigma_y & 0 \\ 0 & 0 & \sigma_z \end{vmatrix} \qquad \mathbf{3}$$

Assuming an $e^{j\omega t}$ time dependent plane wave source and due to the 2D assumption all partial derivatives with respect strike direction (*x*) must be zero in (1):

$$\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} = -j\omega\mu H_x \qquad \mathbf{4}$$

$$\frac{\partial E_x}{\partial z} = -j\omega\mu H_y \qquad \mathbf{5}$$

$$\frac{\partial E_x}{\partial y} = j\omega\mu H_z \qquad \mathbf{6}$$

If we write the component equations of (2), we take into account the conductivity tensor given by (3) and the simplification coming from the zero value of the partial derivatives with respect the strike direction:

$$\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} = \sigma_x E_x \qquad \mathbf{7}$$

$$\frac{\partial H_x}{\partial z} = \sigma_y E_y \qquad \mathbf{8}$$

$$-\frac{\partial H_x}{\partial y} = \sigma_z E_z \qquad \textbf{9}$$

Just like for isotropic 2D MT, there are two independent equation systems. One group consists of equation (4), (8), (9), in which only the strike directional magnetic field component and the non-strike directional electric field components are. This mode is called H-polarization or TM mode. The strike directional magnetic field component is the unknown in the second order partial differential equation, which is the densest form of H-polarization:

$$\frac{1}{\sigma_z}\frac{\partial^2 H_x}{\partial y^2} + \frac{1}{\sigma_y}\frac{\partial^2 H_x}{\partial z^2} = j\omega\mu H_x \qquad \textbf{10}$$

The difference between the isotropic and anisotropic 2D TM modes is obvious: in isotropic case the strike directional magnetic field component depends on the scalar conductivity value and in the anisotropic situation the strike directional magnetic field component depends on both non-strike directional conductivity values.

In the other group of component equations -consisting of (5), (6), (7)- the strike directional electric field component and the non-strike directional magnetic field components can be found. Substituting the value of magnetic field components from (5) and (6) into (7), we receive the E-polarization or TE mode. Similar second order partial differential equation can be derived as (10):

$$\frac{\partial^2 E_x}{\partial y^2} + \frac{\partial^2 E_x}{\partial z^2} = j\omega\mu\sigma_x E_x \qquad \textbf{11}$$

We call the attention for the difference between the 2D TE modes for the anisotropic and isotropic situations: here the strike directional conductivity value, in the isotropic case the scalar conductivity value determines the strike directional electric field component of the second order partial differential equation.

3. 2.5D FEM problem in wave number domain for anisotropic case

3.1 Basic equations

Just like in the case of MT, emphasis is put the Maxwell's equations in the controlled source frequency domain electromagnetic methods. The equation (1) is used in an unchanged form:

$$rot\vec{E} = -\partial\vec{B}/\partial t$$

$\vec{i}_s = Id\vec{s}\,\delta(\vec{r})$ makes possible a point source approximation. An $e^{j\omega t}$ time dependent source term is assumed. Involving it into the other Maxwell's equation of (2):

$$rot\vec{H} = \vec{i} + \vec{i}_s \qquad \textbf{12}$$

It represents the physical law that the magnetic fields are caused by conduction and source currents. The effect of displacement currents in the second equation was not taken into account.

For 2.5D FEM forward modelling with magnetic sources Stoyer and Greenfield recommended that the solution of the original 3D problem could be replaced by the series of solutions of 2D problem in the k_x wave number domain [15]. To follow this procedure the Maxwell's equations (1) and (2) are Fourier-transformed. The Fourier transformation can be expressed as:

$$\hat{F}(k_x, y, z) = \int_{-\infty}^{\infty} F(x, y, z) e^{-jk_x x} dx \qquad \textbf{13}$$

To determine the Fourier transforms of the Maxwell's equations the partial derivative of the EM field components with respect the strike direction (x) is also needed:

$$\int_{-\infty}^{\infty} \frac{\partial F(x,y,z)}{\partial x} e^{-jk_x x} dx = \left[F(x,y,z) e^{-jk_x x}\right]_{-\infty}^{+\infty} + jk_x \int_{-\infty}^{\infty} F(x,y,z) e^{-jk_x x} dx = jk_x \hat{F}(k_x, y, z) \qquad \textbf{14}$$

In (14) equation if x tends to either $+\infty$ or - ∞ then any field component approximates zero due to the finite energy of the source, for this reason the value of expression in brackets must be zero. For this reason the partial derivative of any EM field components with respect the strike direction is proportional the Fourier transform of the component itself. Taking into account this fact and the harmonic time dependence of EM fields as well, the Fourier transforms of Maxwell's equation (1) in terms of component equations can be detailed as follows:

$$\frac{\partial \hat{E}_z}{\partial y} - \frac{\partial \hat{E}_y}{\partial z} = -j\omega\mu \hat{H}_x \quad \mathbf{15}$$

$$\frac{\partial \hat{E}_x}{\partial z} - jk_x \hat{E}_z = -j\omega\mu \hat{H}_y \quad \mathbf{16}$$

$$jk_x \hat{E}_y - \frac{\partial \hat{E}_x}{\partial y} = -j\omega\mu \hat{H}_z \quad \mathbf{17}$$

Due to the TM and TE mode, we have to assume that only horizontal electric dipole source (either in x or y direction) is applied. For these situations we can rewrite the Fourier transforms of (12) as:

$$\frac{\partial \hat{H}_z}{\partial y} - \frac{\partial \hat{H}_y}{\partial z} = \sigma_x \hat{E}_x + \hat{i}_{sx} \quad \mathbf{18}$$

$$\frac{\partial \hat{H}_x}{\partial z} - jk_x \hat{H}_z = \sigma_y \hat{E}_y + \hat{i}_{sy} \quad \mathbf{19}$$

$$jk_x \hat{H}_y - \frac{\partial \hat{H}_x}{\partial y} = \sigma_z \hat{E}_z \quad \mathbf{20}$$

As it was mentioned, the effect of displacement current was abandoned, and for this reason in the isotropic case the complex wavenumber (k) can be expressed as $k^2 = -j\omega\mu\sigma$. Due to the anisotropy we introduce the complex wavenumber in the y and z directions as
$k_y^{\ 2} = -j\omega\mu\sigma_y$ and $k_z^{\ 2} = -j\omega\mu\sigma_z$ It is not necessary to introduce the complex wavenumber in the strike direction, k_x has denoted the strike directional wave number, introduced by the transformation of (14).

Only strike direction EM field components in the wave number (k_x) domain. The first partial differential equation system will describe the situation when the electric dipole source is parallel to the structural strike direction. Another partial differential equation system is valid for the situation when the source term is perpendicular to the structural strike. From equations (16) and (20) $j\hat{H}_y$ can be expressed and taking them to be equal we obtain:

$$\sigma_z (1 - k_x^{\ 2} / k_z^{\ 2}) \hat{E}_z = -\frac{\partial \hat{H}_x}{\partial y} - \frac{k_x}{\omega\mu} \frac{\partial \hat{E}_x}{\partial z} \quad \mathbf{21}$$

With the notation of

$$q_z = 1 / (1 - k_x^{\ 2} / k_z^{\ 2}) \quad \mathbf{22}$$

and with the application of the resistivity instead of conductivity from (21) we receive

$$\hat{E}_z = -\rho_z q_z \frac{\partial \hat{H}_x}{\partial y} - \frac{k_x \rho_z q_z}{\omega\mu} \frac{\partial \hat{E}_x}{\partial z} \quad \mathbf{23}$$

From equations (17) and (19) $j\hat{H}_z$ can be determined and taking them to be equal we obtain:

$$-\sigma_y(1-k_x^{\;2}/k_y^{\;2})\hat{E}_y = -\frac{\partial \hat{H}_x}{\partial z} + \frac{k_x}{\omega\mu}\frac{\partial \hat{E}_x}{\partial y} + \hat{i}_{sy}$$ **24**

If we apply the resistivity instead of the conductivity in (24) and introduce (25) it follows:

$$q_y = 1/(1-k_x^{\;2}/k_y^{\;2})$$ **25**

$$\hat{E}_y = \rho_y q_y \frac{\partial \hat{H}_x}{\partial z} - \frac{k_x \rho_y q_y}{\omega\mu}\frac{\partial \hat{E}_x}{\partial y} - \rho_y q_y \hat{i}_{sy}$$ **26**

We can receive the non-strike directional magnetic field components in the wave number domain similarly. From equations (17) and (19) $\hat{E}_y$ can be determined and we can write:

$$-j\omega\mu(1-k_x^{\;2}/k_y^{\;2})\hat{H}_z = -\frac{\partial \hat{E}_x}{\partial y} + \frac{jk_x}{\sigma_y}\frac{\partial \hat{H}_x}{\partial z} - \frac{jk_x}{\sigma_y}\hat{i}_{sy}$$ **27**

From (27) $\hat{H}_z$ can be expressed:

$$\hat{H}_z = -\frac{jq_y}{\omega\mu}\frac{\partial \hat{E}_x}{\partial y} - \frac{k_x \rho_y q_y}{\omega\mu}\frac{\partial \hat{H}_x}{\partial z} + \frac{k_x \rho_y q_y}{\omega\mu}\hat{i}_{sy}$$ **28**

$\hat{E}_z$ can be determined from equations (16) and (20) and taking them to be equal we receive:

$$j\omega\mu(1-k_x^{\;2}/k_z^{\;2})\hat{H}_y = -\frac{\partial \hat{E}_x}{\partial z} - \frac{jk_x}{\sigma_z}\frac{\partial \hat{H}_x}{\partial y}$$ **29**

From this equation $\hat{H}_y$ can be obtained:

$$\hat{H}_y = \frac{jq_z}{\omega\mu}\frac{\partial \hat{E}_x}{\partial z} - \frac{k_x \rho_z q_z}{\omega\mu}\frac{\partial \hat{H}_x}{\partial y}$$ **30**

3.2 Partial differential equation systems for TE and TM mode of the 2.5D case

The ***TE mode*** is the case when there is only an electric dipole source with strike direction. Substituting equations (28) and (30) into (18) with the assumption of $\hat{i}_{sy} = 0$ in (28) we obtain (31). The other equation for the TE mode is (32) which can be derived by the substitution of (23) and (26) into equation (15) with the assumption of $\hat{i}_{sy} = 0$ in equation of (26):

$$\frac{j}{\omega\mu}\frac{\partial}{\partial y}\left(q_y \frac{\partial \hat{E}_x}{\partial y}\right) + \frac{k_x}{\omega\mu}\frac{\partial}{\partial y}\left(\rho_y q_y \frac{\partial \hat{H}_x}{\partial z}\right) + \frac{j}{\omega\mu}\frac{\partial}{\partial z}\left(q_z \frac{\partial \hat{E}_x}{\partial z}\right) - \frac{k_x}{\omega\mu}\frac{\partial}{\partial z}\left(\rho_z q_z \frac{\partial \hat{H}_x}{\partial y}\right) + \sigma_x \hat{E}_x = -\hat{i}_{sx}$$ **31**

$$\frac{\partial}{\partial y}\left(\rho_z q_z \frac{\partial \hat{H}_x}{\partial y}\right) + \frac{k_x}{\omega\mu}\frac{\partial}{\partial y}\left(\rho_z q_z \frac{\partial \hat{E}_x}{\partial z}\right) + \frac{\partial}{\partial z}\left(\rho_y q_y \frac{\partial \hat{H}_x}{\partial z}\right) - \frac{k_x}{\omega\mu}\frac{\partial}{\partial z}\left(\rho_y q_y \frac{\partial \hat{E}_x}{\partial y}\right) - j\omega\mu \hat{H}_x = 0$$ **32**

In the ***TM mode*** the electric dipole source is perpendicular to the structural strike. In order to receive the TM mode equations similar steps are required as we applied in the TE mode. In this case $\hat{i}_{sx} = 0$ in (18) and $\hat{i}_{sy} \neq 0$ in (28). Substituting (28) and (30) into (18) we obtain equation (33). The second TM mode equation is (34). It can be derived by the substitution of equations (23) and (26) into equation (15):

$$\frac{j}{\omega\mu}\frac{\partial}{\partial y}\left(q_y\frac{\partial \hat{E}_x}{\partial y}\right)+\frac{k_x}{\omega\mu}\frac{\partial}{\partial y}\left(\rho_y q_y\frac{\partial \hat{H}_x}{\partial z}\right)+\frac{j}{\omega\mu}\frac{\partial}{\partial z}\left(q_z\frac{\partial \hat{E}_x}{\partial z}\right)-\frac{k_x}{\omega\mu}\frac{\partial}{\partial z}\left(\rho_z q_z\frac{\partial \hat{H}_x}{\partial y}\right)+\sigma_x\hat{E}_x=\frac{k_x}{\omega\mu}\frac{\partial}{\partial y}\rho_y q_y\hat{i}_{sy} \qquad \mathbf{33}$$

$$\frac{\partial}{\partial y}\left(\rho_z q_z\frac{\partial \hat{H}_x}{\partial y}\right)+\frac{k_x}{\omega\mu}\frac{\partial}{\partial y}\left(\rho_z q_z\frac{\partial \hat{E}_x}{\partial z}\right)+\frac{\partial}{\partial z}\left(\rho_y q_y\frac{\partial \hat{H}_x}{\partial z}\right)-\frac{k_x}{\omega\mu}\frac{\partial}{\partial z}\left(\rho_y q_y\frac{\partial \hat{E}_x}{\partial y}\right)-j\omega\mu\hat{H}_x=\frac{\partial}{\partial z}\rho_y q_y\hat{i}_{sy} \qquad \mathbf{34}$$

Just like in the case of isotropic 2.5D case, both modes can be described with the same partial differential equations and the differences between the modes are presented by the inhomogeneous terms on the right. The second order partial derivatives with respect to *y* or *z* can be finite differenced similarly as in the case of isotropic situation, but the terms with second order mixed partial derivatives should be transformed before they are finite differenced. For example instead of the first mixed second derivative of the strike directional Fourier magnetic field component in equation (31) or in (33) can be rewritten as:

$$\frac{k_x}{\omega\mu}\frac{\partial}{\partial y}\left(\rho_y q_y\frac{\partial \hat{H}_x}{\partial z}\right)=\frac{k_x}{\omega\mu}\frac{\partial(\rho_y q_y)}{\partial y}\frac{\partial \hat{H}_x}{\partial z}+\frac{k_x\rho_y q_y}{\omega\mu}\frac{\partial^2\hat{H}_x}{\partial y\partial z} \qquad \mathbf{35}$$

The next mixed second order partial derivative of the strike directional Fourier magnetic field components in (31) or in (33) can be transformed similarly:

$$\frac{k_x}{\omega\mu}\frac{\partial}{\partial z}\left(\rho_z q_z\frac{\partial \hat{H}_x}{\partial y}\right)=\frac{k_x}{\omega\mu}\frac{\partial(\rho_z q_z)}{\partial z}\frac{\partial \hat{H}_x}{\partial y}+\frac{k_x\rho_z q_z}{\omega\mu}\frac{\partial^2\hat{H}_x}{\partial y\partial z} \qquad \mathbf{36}$$

Except for the coefficient of the strike directional Fourier magnetic field component in (32) or in (34) all coefficients on the left hand sides reflect the effect of anisotropy. The effect of anisotropy can be observed even in the inhomogeneous terms in equations of (33) and (34) in the TM mode. The latter statement does not hold for the TE mode. In contrast with the isotropic situation [3], second order mixed partial derivatives of the strike directional Fourier EM field components are in the anisotropic case.

3.3 Determination of spatial solutions

In order to obtain space-domain solution from the solution received in the strike-directional wavenumer domain, inverse Fourier transforms have to be applied. The relationship resulting in space-domain solution from the wave number domain is as follows:

$$F(x,y,z)=\frac{1}{2\pi}\int_{-\infty}^{\infty}\hat{F}(k_x,y,z)e^{jk_x x}dk_x \qquad \mathbf{37}$$

For the ***TE mode*** it can be proved [3] that in broadside configuration only E_x, H_y, H_z components are different from zero. By inverse transform of (37) for the vertical plane containing the source (x=0) the strike directional electric field component for any point can be received as:

$$E_x(0,y,z)=\frac{1}{\pi}\int_0^{\infty}\hat{E}_x(k_x,y,z)\,dk_x \qquad \mathbf{38}$$

The non-strike directional magnetic field components can be determined from equations (30) and (28) with the assumption of source free terms. For the vertical plane containing the source (x=0) the spatial magnetic components can be gained based upon equation (37):

$$H_y(0,y,z)=\frac{1}{\pi}\int_0^{\infty}\left[\frac{jq_z}{\omega\mu}\frac{\partial \hat{E}_x}{\partial z}-\frac{k_x\rho_z q_z}{\omega\mu}\frac{\partial \hat{H}_x}{\partial y}\right]dk_x \qquad \mathbf{39}$$

$$H_z(0,y,z) = -\frac{1}{\pi}\int_0^{\infty} [\ \frac{jq_y}{\omega\mu}\frac{\partial \hat{E}_x}{\partial y} + \frac{k_x\rho_y q_y}{\omega\mu}\frac{\partial \hat{H}_x}{\partial z}\ ]dk_x \qquad \textbf{40}$$

For the ***TM mode*** in collinear configuration the following three electromagnetic field components - E_y, E_z, H_x - are not zero and they can similarly be expressed as the other three field components for TE mode. The expression for the strike directional magnetic field component is an inverse Fourier-transform of the wave number solution:

$$H_x(0,y,z) = \frac{1}{\pi}\int_0^{\infty} \hat{H}_x(k_x,y,z)\, dk_x \qquad \textbf{41}$$

With the assumption of source free region and with the application of (37), the Fourier-transform of non-strike directional electric field components can be determined from (26) and (23)

$$E_y(0,y,z) = \frac{1}{\pi}\int_0^{\infty} [\ \rho_y q_y \frac{\partial \hat{H}_x}{\partial z} - \frac{k_x\rho_y q_y}{\omega\mu}\frac{\partial \hat{E}_x}{\partial y}\ ]dk_x \qquad \textbf{42}$$

$$E_z(0,y,z) = -\frac{1}{\pi}\int_0^{\infty} [\ \rho_z q_z \frac{\partial \hat{H}_x}{\partial y} + \frac{k_x\rho_z q_z}{\omega\mu}\frac{\partial \hat{E}_x}{\partial z}\ ]dk_x \qquad \textbf{43}$$

It can be noted that there is no difference between the inverse Fourier formulae of the strike directional EM field components valid for the anisotropic (38), (41) and the isotropic case [3]. At the same time in the course of the determination of the secondary EM field components additional anisotropy effect is superimposed to the primary EM field components. For this special case of anisotropy the conductivity perpendicular to the strike and the conductivity value in the *z* direction have effect on the secondary EM field components (39), (40), (42), (43).

4.Comparison between FEM 2D and 2.5D equations in the point of view of conductivity anisotropy

In the 2.5D FEM there are two unknowns in the two coupled equations, they are the Fourier transforms of the strike directional electric and magnetic field components. Due to the differential equation systems (consisting of (31) and (32) or (33) and (34)) there are neither pure TE nor TM mode, i.e. the two modes are coupled. The three anisotropy values have effect on the strike directional EM field components. The anisotropy effect is controlled by the three principal values of conductivity in both modes, i.e. the three anisotropy values have effect on the strike directional EM field components. For isotropic case the partial differential equations do not contain mixed second order derivatives and one geoelectric domain can be characterized by a constant electrical conductivity value.

In 2D MT it is equation (11) which governs the TE mode EM fields' behaviors. In the TE mode partial differential equation the strike directional electric field component is the only unknown and the strike directional conductivity value controls the effect of anisotropy. In the geophysical EM special emphasis is put on CSAMT (Controlled Source Audio Magnetotellurics) and VLF(Very Low Frequency) methods, both are working in the far-field zone. The theoretical far field zone (corresponding to MT) is presented from these equations if $k_x=0$. For this strike directional wave number value $q_z=q_y=1$ and assuming $\hat{i}_{sy}=0$ in (31) we can easily receive from (31):

$$\frac{\partial}{\partial y}\left(\frac{\partial \hat{E}_x}{\partial y}\right) + \frac{\partial}{\partial z}\left(\frac{\partial \hat{E}_x}{\partial z}\right) = j\omega\mu\sigma_x \hat{E}_x \qquad \textbf{44}$$

The similarity between (11) and (44) is obvious. We can state that if we approach the theoretical far field zone, not only the coupling has been ceased, but the effect of the non-strike directional conductivities can be abandoned as well. Taking into account that the second term of integrand is zero and $q_z=q_y=1$ in equations (39) and (40), we receive similar physical relationship for the non strike directional magnetic field components of TE mode as we can have from (5)

$$H_y = \frac{j}{\omega\mu}\frac{\partial E_x}{\partial z} \quad \textbf{45}$$

and from (6):

$$H_z = -\frac{j}{\omega\mu}\frac{\partial E_x}{\partial y} \quad \textbf{46}$$

As for the 2.5D theoretical far field TM mode approximation is concerned instead of (32) -with the same assumption as we applied in the case of TE mode- we can write:

$$\frac{\partial}{\partial y}\left(\rho_z \frac{\partial \hat{H}_x}{\partial y}\right) + \frac{\partial}{\partial z}\left(\rho_y \frac{\partial \hat{H}_x}{\partial z}\right) = j\omega\mu\hat{H}_x \quad \textbf{47}$$

We can state that if we approach the theoretical far field zone over a 2D anisotropic conductivity structure in the TM mode, the effect of the non strike directional conductivity is dominant over that of the strike directional conductivity (i.e. the conductivities in dip and the vertical direction control the effect of anisotropy). In this zone the similarity between the controlled source and MT non strike directional electric field components is obvious.
In order to make comparison between them from (8)

$$E_y = \rho_y \frac{\partial H_x}{\partial z} \quad \textbf{48}$$

and from (9):

$$E_z = -\rho_z \frac{\partial H_x}{\partial y} \quad \textbf{49}$$

Because the second term of integrand is zero and $q_z = q_y = 1$ in equations (42) and (43), we received similar relationship for the non-strike directional electric field components of the TM mode for the two cases.

5. Summary

The effect of anisotropy cannot be abandoned in the course of EM exploration. In the paper the basic equations were derived for elongated anisotropic conductivity structures exited by plane wave and electric dipole sources (treated as point sources, with either in strike or dip direction). Based upon these equations only after the discretization of them it is possible to model the EM responses over some groups of frequency domain electromagnetic methods.
It was proven that in the 2D MT TE (E polarization) mode partial differential equation the strike directional field component is the unknown and the strike directional conductivity value controls the effect of anisotropy. Similarly, in the 2D MT TM (H polarization) mode equation the strike directional magnetic field component is the unknown and the conductivities in dip and the vertical direction control the effect of anisotropy. In the course of the 2.5D anisotropic FEM problem two partial differential equations in the strike directional wave number domain were derived and the two polarizations can be distinguished by the inhomogeneous terms. The form of the coupled partial differential equations is more complex than for isotropic case because of the presence of the second order mixed partial derivatives. The unknowns of the differential equations are the Fourier transforms of the strike directional electric and magnetic field components and in contrast with anisotropic 2D MT they are simultaneously controlled by the three principal values of the conductivity tensor. In general case - including the transition zone- additional anisotropy effect (except the effect of the strike directional conductivity) is superimposed when the secondary EM field components are determined. Approaching the theoretical far field zone similarity can be observed between the EM field components obtained by the controlled and the plane wave source methods in the point of view of anisotropy effect.

NOMENCLATURE

$\vec{B}$: magnetic flux density (Wb/m^2)
$d\vec{s}$: elementary dipole length (m)
$\vec{E}$: electric field vector (V/m)
F: EM component (V/m, or A/m)
$\hat{F}$: Fourier-transform of the F EM component (V/m, or A/m)
$\vec{H}$: magnetic field vector (A/m)
I: transmitter current (A)
$\vec{i}$: current density vector (A/m^2)
$\vec{i}_s$: applied current source (A/m^2)
j : $\sqrt{-1}$
x: strike direction
k: complex wavenumber (1/m)
k_x: wavenumber in the strike direction (1/m)
p: skin depth (m)
q_z: derived dimensionless parameters (subscript z refers to vertical)
q_y: derived dimensionless parameters (subscript y refers direction perpendicular to the structural strike)
r: transmitter-receiver separation, distance (m)
t: time (s)
$\delta(r)$: Dirac-delta function
ε: dielectric constant (F/m)
μ: magnetic permeability (H/m)
ρ: resistivity (ohmm)
σ: conductivity (mho/m or S/m)
ω: angular frequency of the EM field (Hz)

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REFERENCES

Takács, E.: "Geophysics, Geoelectrical exploration methods I., II." (in Hungarian), Budapest, Tankönyvkiadó, 1981.
Nabighien, M. N.: "Electromagnetic methods in applied geophysics." Vol.2, Application, Parts A and B, Tulsa, SEG, 1992.
Pethő, G.: "FEM numerical modeling of 2D structures exited by electrical dipole source." (in Hungarian, thesis for Candidate of Science degree), Miskolc, 1993.
Pethő, G.: "CSAMT Numerical Modelling for 2D Thermal EOR Monitoring.", EAPG-6th Conference and Technical Exhibition Extended Abstracts of Papers, P547, Vienna, 1994.

Pethő, G., Kaikkonen, P., Vanyan, L., L.: "Numerical modelling for the effect of a 2-D seafloor trench on sea-bottom EM measurements using horizontal electric dipole sources." Geophysica, Vol. 31., 1995, 1-21.

Pethő, G., Kaikkonen, P., Vanyan, L., L.: "Frequency domain electromagnetic source polarization investigation in case of a 2-D seafloor trench." Aarhus, 3rd European Environmental and Engineering Geophysics Conference, Section New Developments, 1997, 323-326.

Pethő, G., Ficsór, L.: "Distortion effect of elongated surface conductivity-inhomogeneities on CSAMT measurements." 14th Workshop on EM induction in the Earth, Book of Abstract, p. 100, Sinaia, 1998.

Pethő, G., Ficsór, L.: "Source polarization effect in case of elongated surface inhomogeneities covering transition zone." Amsterdam, EAGE 63rd Conference & Technical exhibition, Abstract II, P050, 2001.

Pethő, G.: "FEM source effect investigation with 2.5D numerical modeling." 15th European Meeting of Environmental and Engineering Geophysics, Dublin, Near Surface Proceedings &Exhibitors'Cataloge, P59, 2009.

Pethő, G.: "Differential equations of FEM using electric dipole source for elongated structures with conductivity anisotropy." Kutatás és Innováció a Magyar Geotermiában, konferencia kiadvány, pp.70-86. , ISBN 978-963-358-005-9, Miskolc, 2012.

Pethő, G.: "Bases of FD modeling for EM underground transillumination with vertical electric dipoles in 2D anisotropic conductivity structures." Geosciences and Engineering. Vol. 2, No.3., pp.51-62., 2013.

Pethő, G., Ormos, T., Turai, E., Szabó, N., Bulla, D., Németh, N., Zergi, I., Benő, D.,Kocsis, S.: " Geophysical measurements in the vicinity of Bükkszentkereszt." (in Hungarian), Bányászati és Kohászati Lapok- Bányászat 146:(5-6) pp. 39-47., 2013.

Keller, G.,V.: "Rock and Mineral Properties." in: Nabighien, M. N.: "Electromagnetic methods in applied geophysics." Vol.1, Theory, Tulsa, SEG, 1992.

Negi, J., G., Saraf, P.,D.: "Anisotropy in geoelectromagnetism." Methods in geochemistry and geophysics, 28., Elsevier, 1989.

Stoyer, C., H., Greenfield, R., J.: "Numerical solutions of the response of a two-dimensional earth to an oscillating magnetic dipole source." Geophysics, 41, 1976, 519-530.

Tartalomjegyzék

APPENDIX 1: **A telegráf egyenlet levezetése**

A kiindulás a Maxwell egyenletek differenciális alakja:

$$rot\vec{H} = \vec{j} + \frac{\partial \vec{D}}{\partial t} \qquad (1)$$

$$rot\vec{E} = -\frac{\partial \vec{B}}{\partial t} \qquad (2)$$

Az első Maxwell egyenlet szerint mind a vezetési mind az eltolási áram mágneses örvényteret hoz létre. A második alapján az elektromos tér örvényerőssége arányos a mágneses indukció időegységre eső változásával. A fenti összefüggésekhez tartozó kiegészítő egyenletek a következők:

$$div\vec{D} = \rho_v \qquad (3)$$

$$div\vec{B} = 0 \qquad (4)$$

Azaz az elektromos eltolódás forrásbősége arányos a (térfogati) töltéssűrűséggel (3) alapján, ill. a mágneses tér forrásmentes, mert nincsenek szétválasztható mágneses töltések (4) szerint. Az egyenletekben szereplő vektor mennyiségek nem függetlenek egymástól. Köztük a lineáris összefüggés a legegyszerűbb feltételezés, amely homogén, izotróp estre vonatkozó közelítés. Ezen idealizált estre megfogalmazott anyagi egyenletek az alábbiak:

$$\vec{j} = \sigma\vec{E} \qquad \vec{D} = \varepsilon\vec{E} \qquad \vec{B} = \mu\vec{H} \qquad (5\text{-}7)$$

A vezetési áraműrűséget az elektromos térerősség határozza meg (differenciális Ohm-törvény) ahol az arányossági tényező a fajlagos vezetőképesség; az elektomos eltolódás és az elektromos térerősség közötti lineáris összefüggésben az arányossági tényező a közeg dielektromos állandója; míg az utolsó egyenlet szerint a mágneses indukciót a mágneses térerősség határozza meg a közeg mágneses permeabilitásától függően.

Tételezzük fel az elektromágneses térerősségek $e^{i\omega t}$ szerinti harmonikus időfüggését, továbbá azt, hogy nincsenek szabad elektromos töltések a σ fajlagos vezetőképességű tértartományban. Így a térerősség vektorok idő szerinti deriváltjai helyett a térerősség vektorok (iω)-szorosa vehető, másrészt a (6) anyagi egyenlet szerint nemcsak az elektromos eltolódás vektorának (3), hanem az elektromos térerősség vektorának divergenciája is zérus lesz. Az eddigiek figyelembevételével (1), ill. (2) helyett írható:

$$rot\vec{H} = \sigma\vec{E} + i\varpi\varepsilon\vec{E} \qquad (8)$$

$$rot\vec{E} = -i\varpi\mu\vec{H} \qquad (9)$$

Képezzük a (9) alakú, második Maxwell egyenlet rotációját:

$$rotrot\vec{E} = -i\varpi\mu rot\vec{H} \qquad (10)$$

A (10) bal oldalán lévő mennyiség meghatározható az alábbi vektorazonosság alapján is:

$$rotrot\vec{E} = graddiv\vec{E} - \Delta\vec{E} \qquad (11)$$

Így (10) és (11) jobb oldalai is egyenlők egymással. Annak érdekében, hogy csak az elektromos térerősség szerepeljen, a (10) jobb oldalán (8) szerinti helyettesítést végezzük el, míg (11) jobb oldalán vegyük figyelembe, hogy az elektromos térerősség divergenciája zérus:

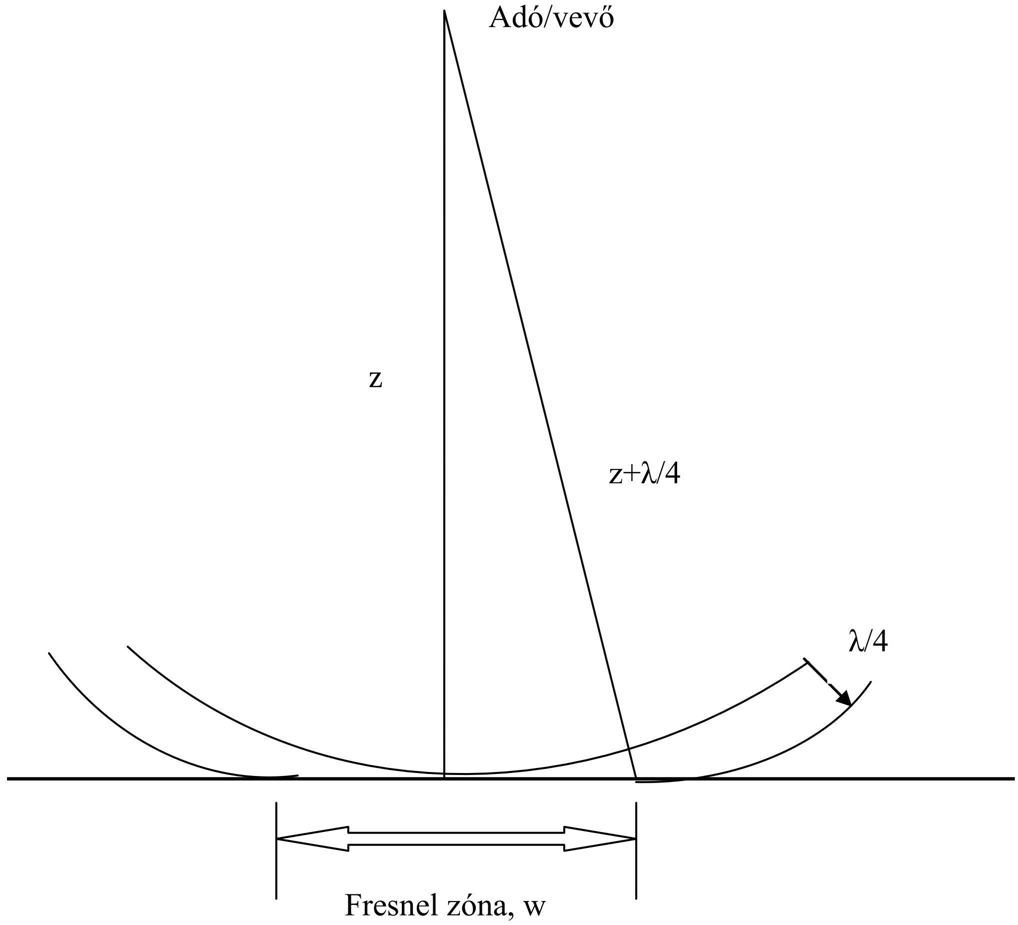
$$-i\varpi\mu rot\vec{H} = -i\varpi\mu\left(\sigma\vec{E} + i\varpi\varepsilon\vec{E}\right) = graddiv\vec{E} - \Delta\vec{E} = -\Delta\vec{E} \qquad (12)$$

Ezen egyenlet így a következő alakú lesz:

$$\Delta\vec{E} + \left(\mu\varepsilon\varpi^2 - i\varpi\mu\sigma\right)\vec{E} = \Delta\vec{E} + k^2\vec{E} = \vec{0} \qquad (13)$$

Formailag teljesen hasonló alakú egyenlet vonatkozik a mágneses térerősség vektorra.
Ez a (8) egyenlet rotációjának képzésével, majd (9) (8)-nak jobb oldali helyettesítésével érhető el a mágneses térerősségre vonatkozó (11) vektor azonosság feltételezése mellett.
A végeredményt felírva: $\Delta\vec{H} + \left(\mu\varepsilon\varpi^2 - i\varpi\mu\sigma\right)\vec{H} = \Delta\vec{H} + k^2\vec{H} = \vec{0}$ (14)

APPENDIX 2: **Fresnel-zóna**



Az a reflektált energia, amely fél hullámhossznál kisebb fáziskéséssel érkezik be, mint az első beérkezésű reflektált jel, az a reflexiót erősíti, ezt nevezzük konstruktív interferenciának. Ezt a felületet Fresnel zónának nevezzük. A w-nél kisebb horizontális felületről kapott reflexiókat nem lehet elkülöníteni. Az ábrán látható derékszögű háromszögre írható, hogy

$$z^2 + (w/2)^2 = (z + \lambda/4)^2$$

Négyzetre emelést követően (λ^2/16)-ot a bal oldalon elhanyagolva kapjuk, hogy:

$$w = \sqrt{2\lambda z}$$

Adott mélységben tehát annál kisebb a Fresnel-zóna, minél nagyobb a frekvencia, azaz a horizontális felbontás mértéke a frekvencia növelésével fokozható. Ugyanakkor a kimutathatóság a vizsgált objektum mélységétől is függ, rögzített frekvencia mellett a mélység növelésével a kimutatathatóság csökken.